Investigating Climate Change Using Mathematical Modeling

Is climate change real? Based on recorded historical data

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MATH 350 Mathematics Modeling

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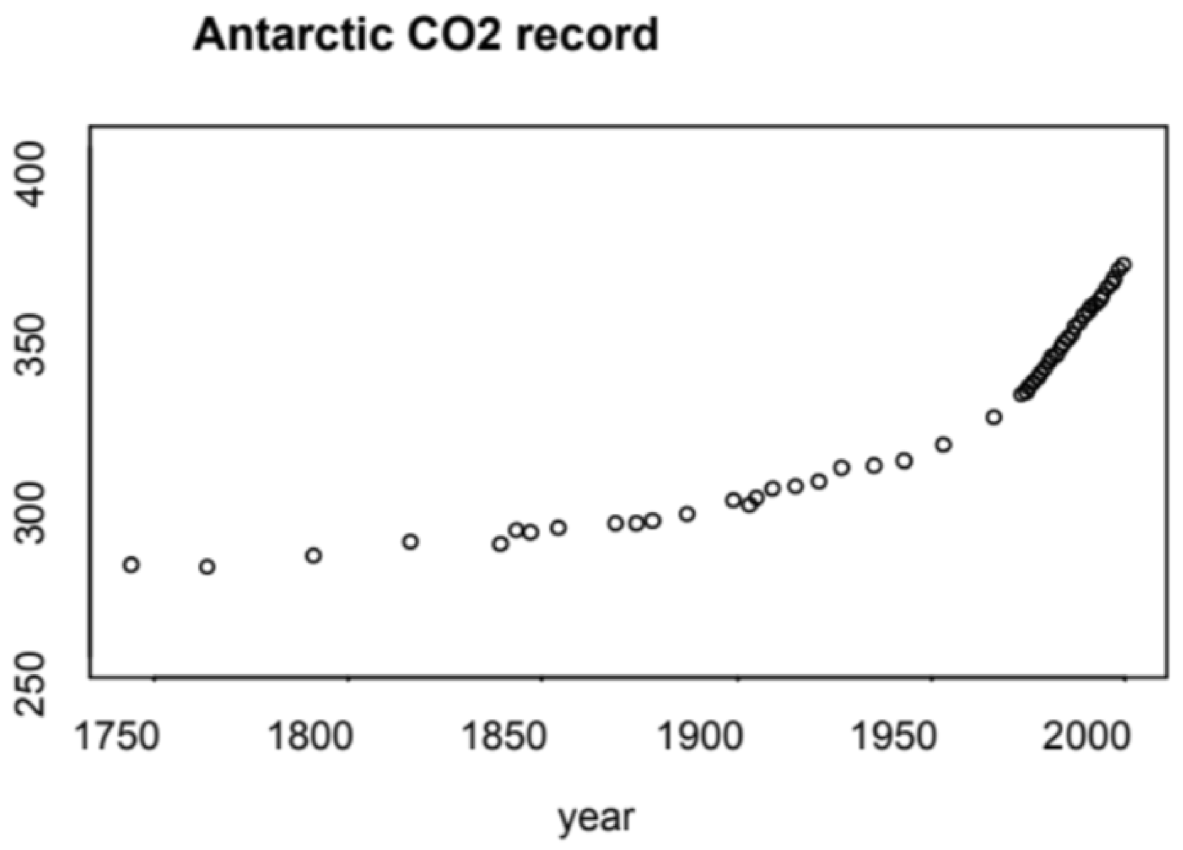
Background

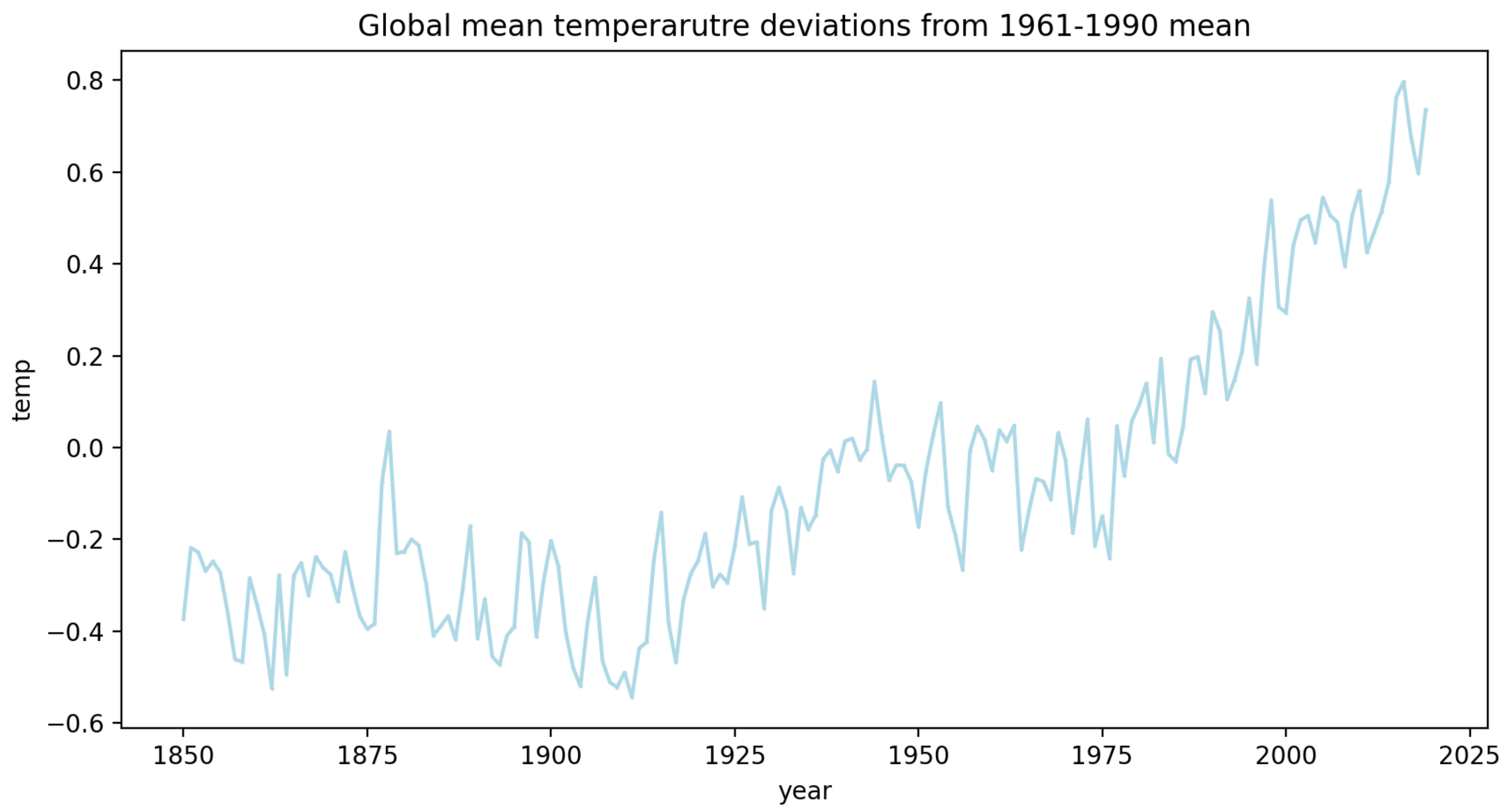
- Climate Change in general and what we already known
- Human activities cause the rise of CO_2 Concentration in the atmosphere.
- Correlation between CO_2 and rising temperature ?

Real Data (*)



Antarctic CO2 record
400
350
300
250
1750
1800
1850
1900
1950
2000
year

Real Data

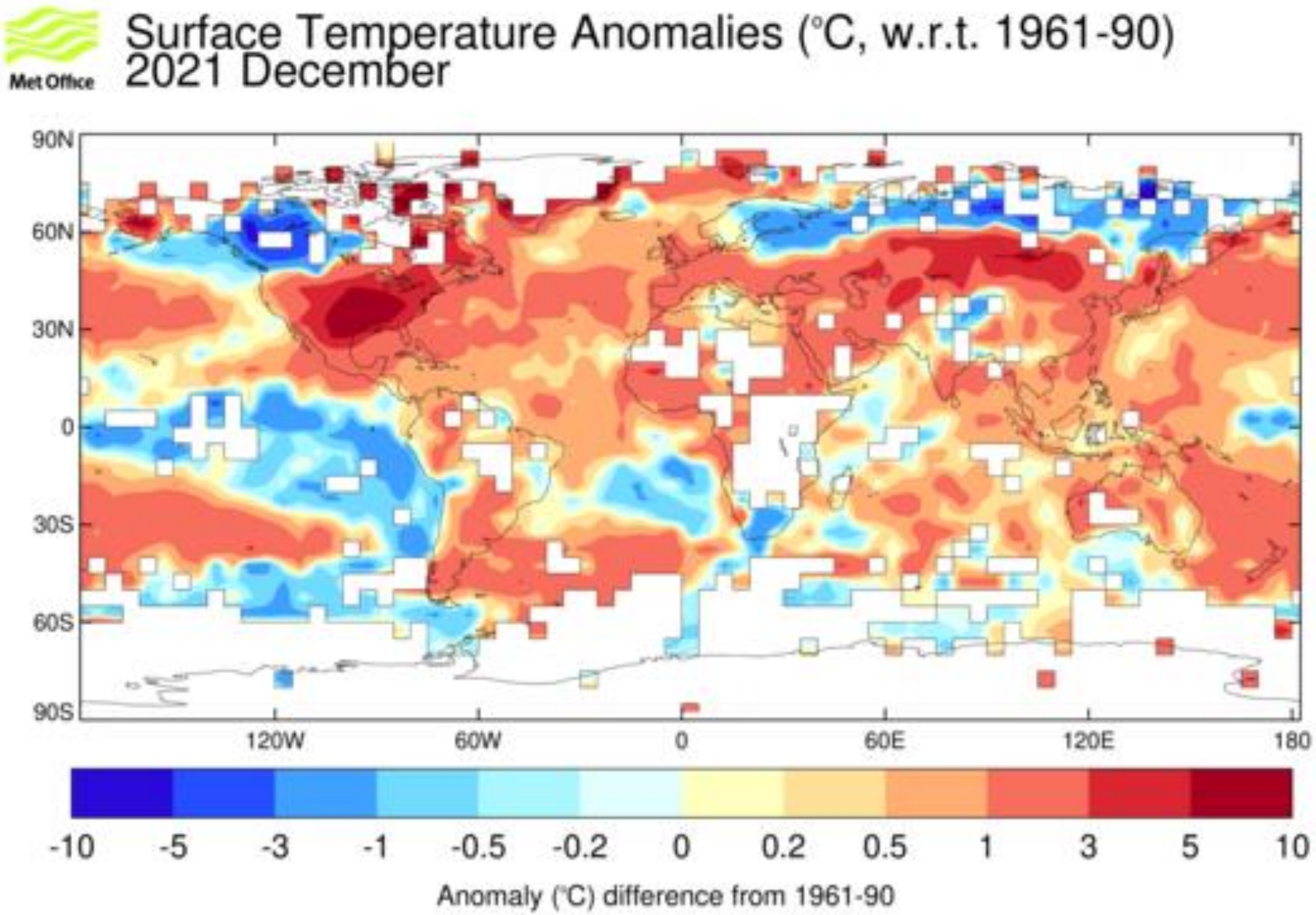


Global mean temperarutre deviations from 1961-1990 mean
temp
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
1850
1875
1900
1925
1950
1975
2000
2025
year

Where is the Real Data from?

Met Office Hadley Centre observations datasets

Dataset: HadCRUT5



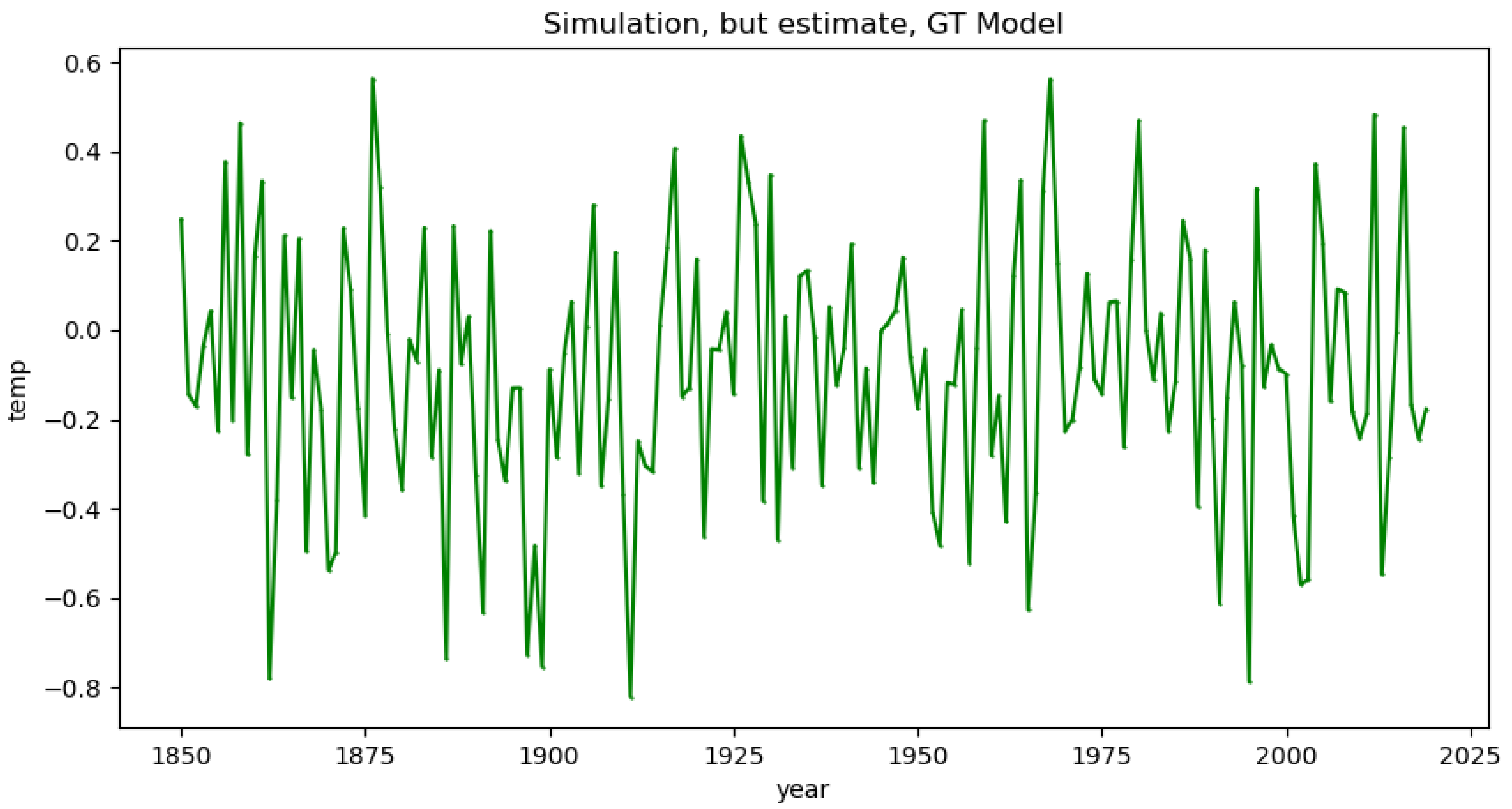
First Model

- $T_i = \alpha + E_i, E_i \sim N(0, \sigma^2)$
- $\alpha = mean\ temperature$
- $\sigma = \sqrt{temp\ variance}$

$\alpha \sim -0.0085$

$\sigma \sim 0.3$

```
alpha = stat.mean(temp)
mu, sigma = 0, stat.variance(temp)**0.5
e = np.random.normal(mu,sigma, 170) #170 = 2019 - 1850 + 1
temp = alpha + e
```


Simulation, but estimate, GT Model
temp
year
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
1850
1875
1900
1925
1950
1975
2000
2025

Comparing to Real Data

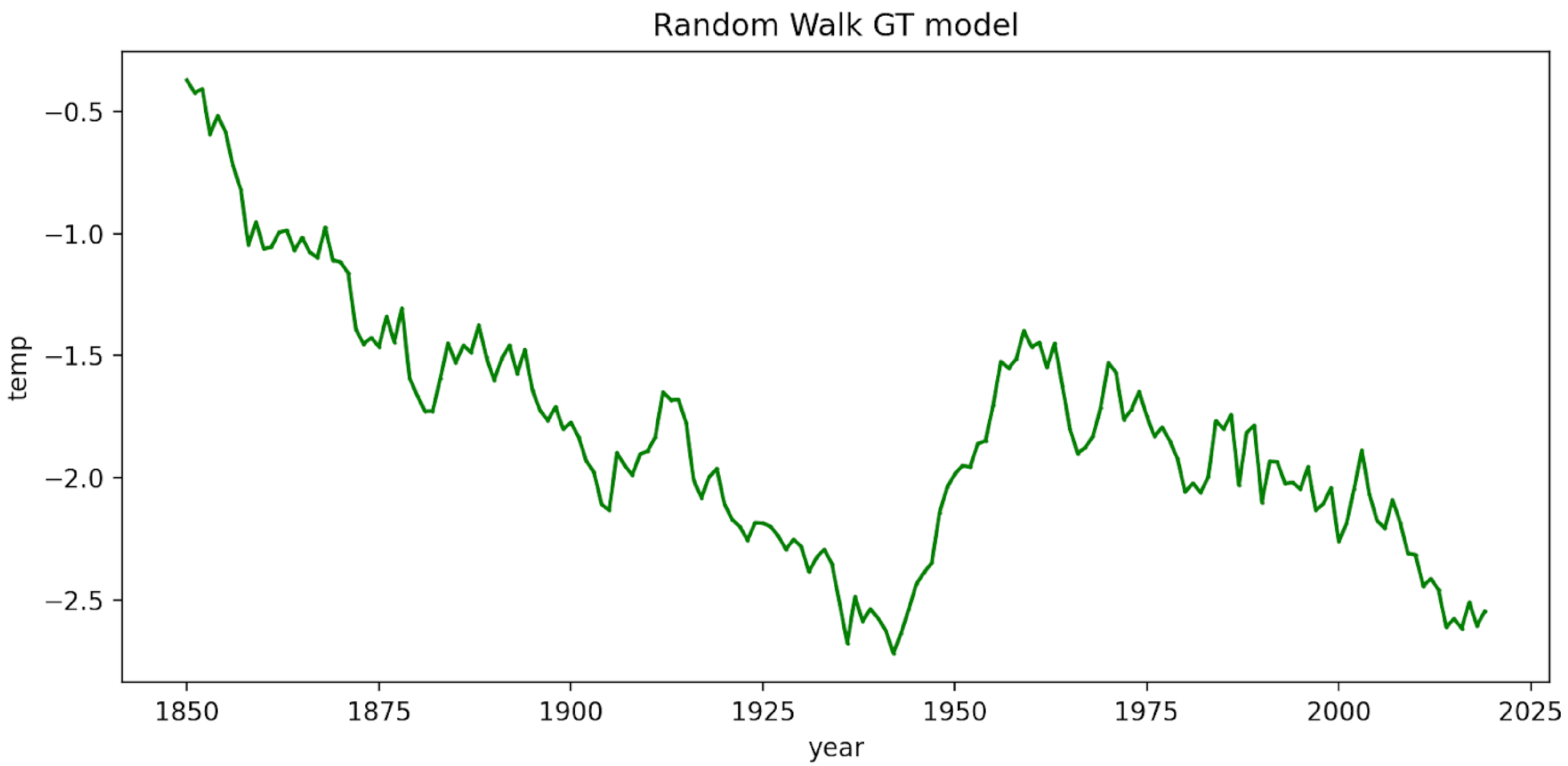
- Different from Real data.
- Produce too much year-to-year variation
- Need an alternative model

Second Model

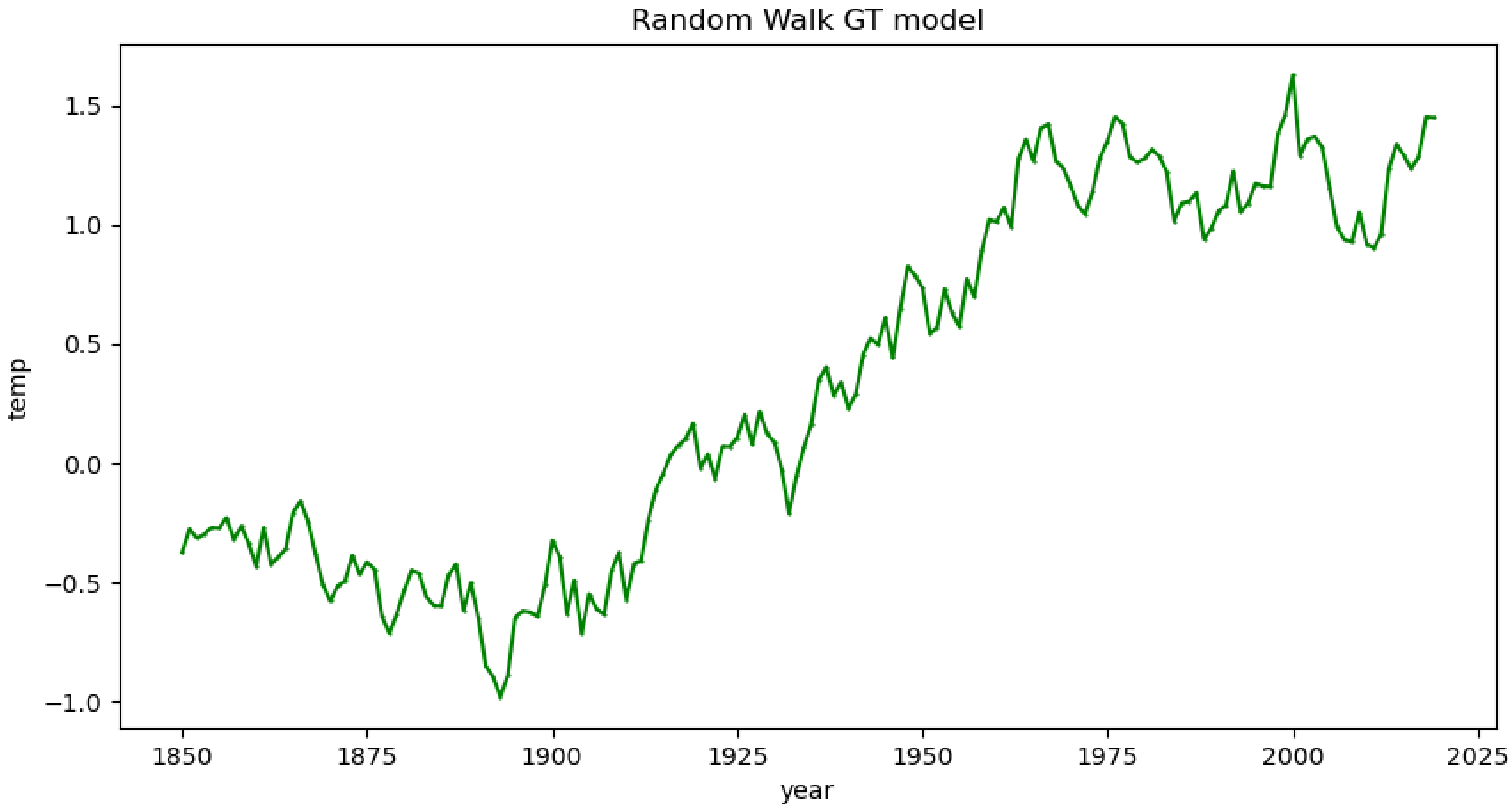
- *Random Walk*
- $T_{i+1} = T_i + E_i,\ E_i \sim N(0, \sigma^2)$
- $\sigma = \sqrt{\Delta T}\ (\Delta T = T_{i+1} - T_i)$

$\sigma \sim 0.12$

```
diff = np.empty(0)
for i in range(y.size-1):
  diff = np.append(diff, y[i+1] - y[i])

sigma = stat.variance(diff)**0.5

temp = np.empty(0)
temp = np.append(temp,y[0])

e = np.random.normal(0,sigma, 170)

for i in range(1, e.size):
  temp = np.append(temp, temp[i-1]+e[i-1])
```



Random Walk GT model
temp
−0.5
−1.0
−1.5
−2.0
−2.5
1850
1875
1900
1925
1950
1975
2000
2025
year


Random Walk GT model
temp
year
1.5
1.0
0.5
0.0
−0.5
−1.0
1850
1875
1900
1925
1950
1975
2000
2025

Compare

- First Model: too much variation
- Second Model: too little year to year variation
- Unbounded and walk to any end.

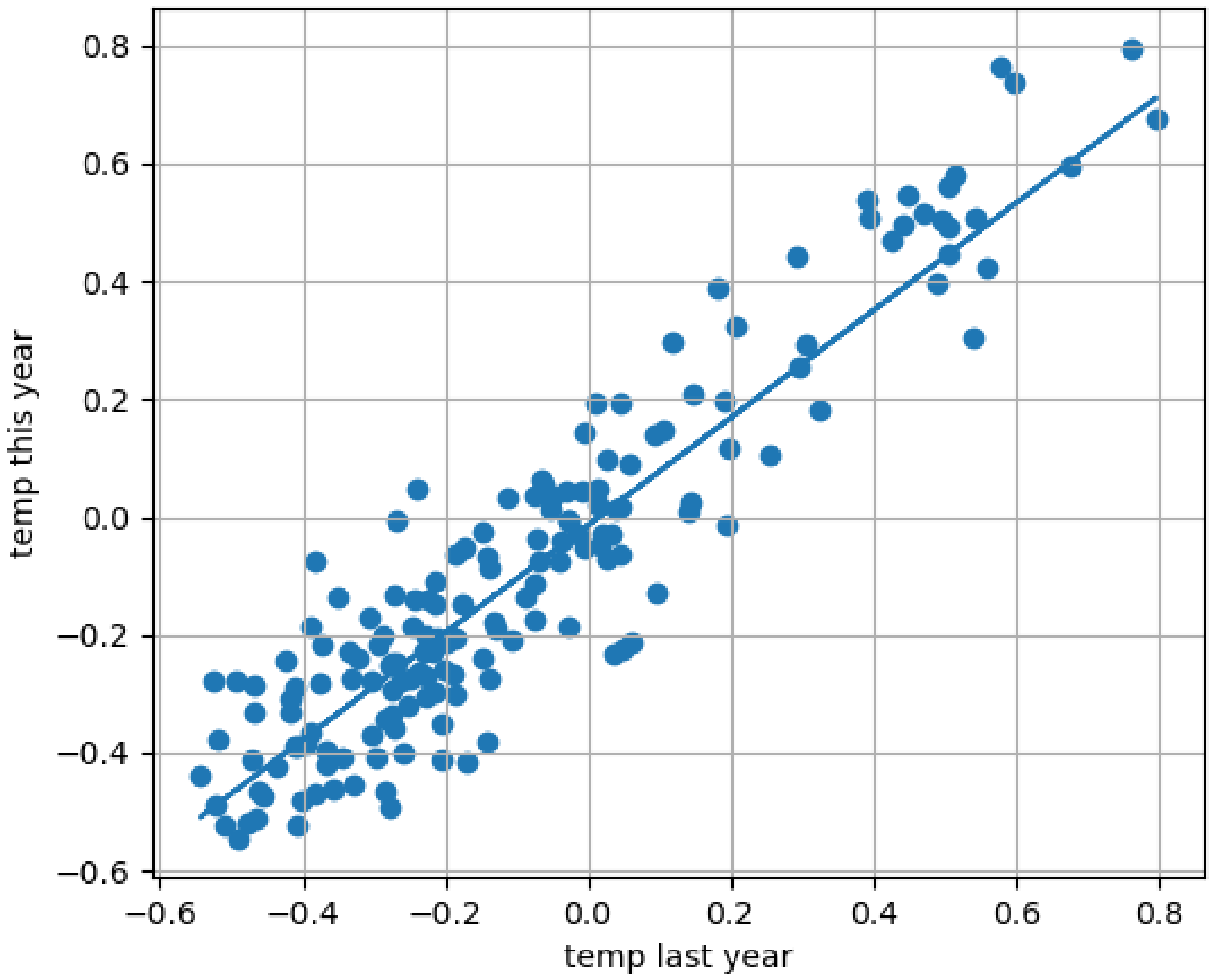
- We need a model between the First and the Second Model
- Using Auto-regressive method

Third Model

- *Auto-regressive*
- $T_{i+1} = \alpha + \beta T_i + E_i$

```
t0 = y[1:170]
t1 = y[0:169]

beta, alpha = np.polyfit(t0,t1,1)
sigma_0 = stat.variance(t0-alpha-beta*t1)
```

$$f(x) = -0.01407 + 0.91062x$$



Third Model

- *Auto-regressive*
- $T_{i+1} = \alpha + \beta T_i + E_i$

```
t0 = y[1:170]
t1 = y[0:169]

beta, alpha = np.polyfit(t0,t1,1)
sigma_0 = stat.variance(t0-alpha-beta*t1)
```

- $\alpha \sim \ -0.014,\ \ \beta \sim 0.91$
- $\sigma \sim 0.013$

But how close is beta and alpha?

- Since we use np.polyfit and it doesn’t ensure us what method it used to compute alpha, beta.
- To make sure, we need to use least square estimation

Third Model

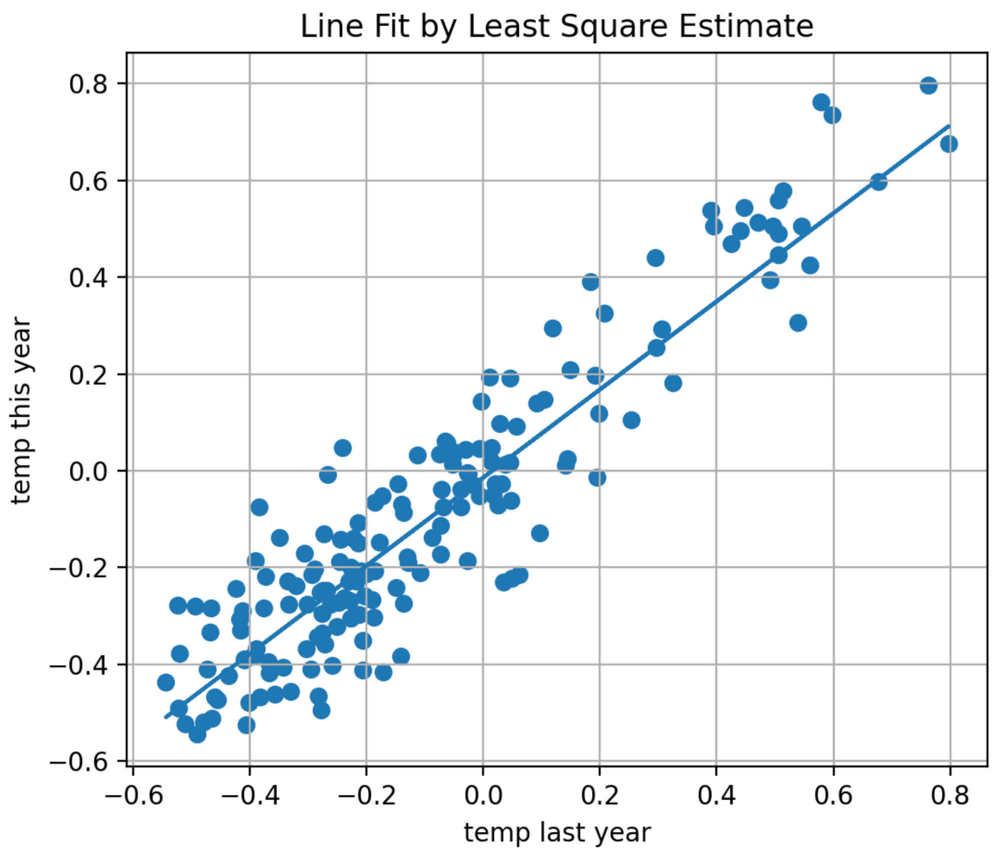
- *Auto-regressive*
- $T_{i+1} = \alpha + \beta T_i + E_i$
- *Least square method*

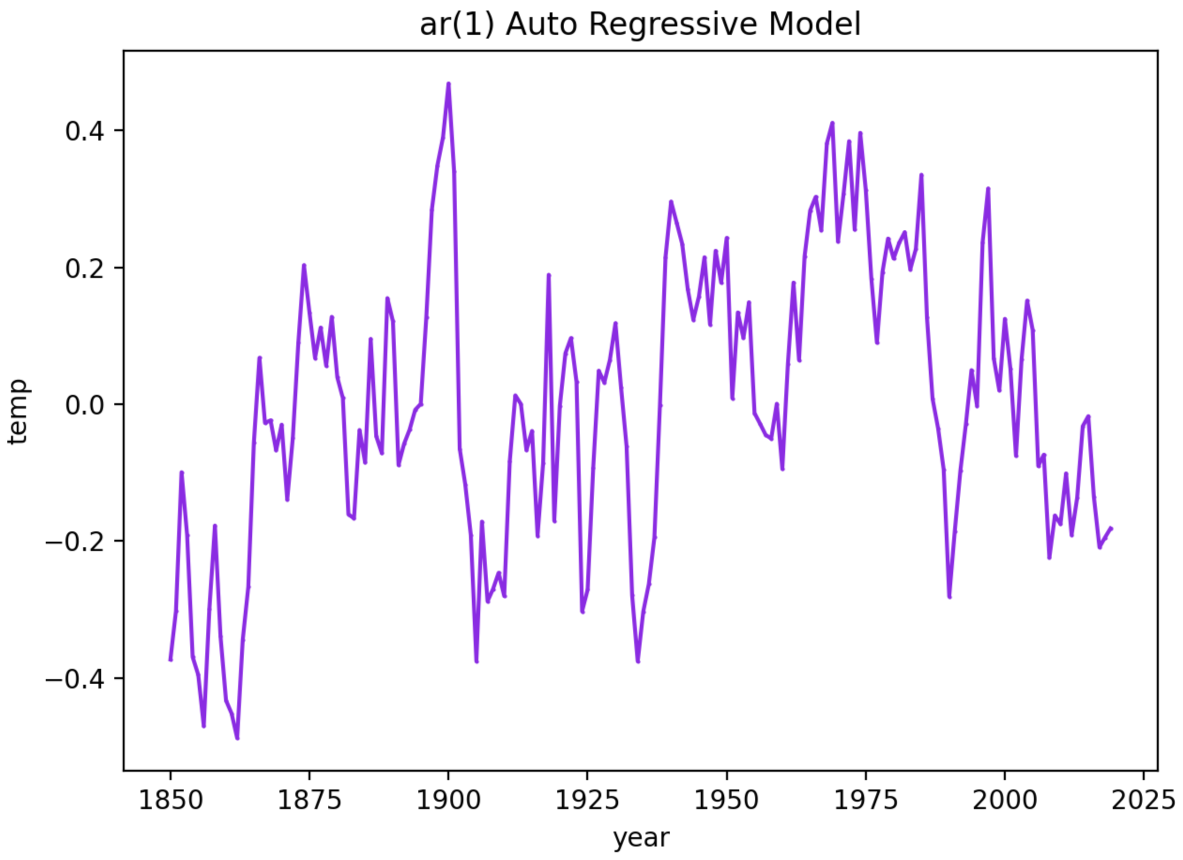
$$\begin{vmatrix} \mathbf{1} & \mathbf{T0} \\ 1 & T1 \\ & \dots \\ 1 & Tn\text{-}1 \end{vmatrix} \begin{vmatrix} \mathbf{alpha} \\ beta \\ \\ \\ \end{vmatrix} = \begin{vmatrix} \mathbf{T1} \\ T2 \\ \dots \\ Tn \end{vmatrix}$$

$$\alpha \sim -0.014,\ \beta \sim 0.91, \sigma \sim 0.013$$

```
def leastSquare(temp1, temp2):
    matrixA = np.zeros((len(temp1),2))
    matrixB = temp1

    for i in range(len(temp1)):
        matrixA[i][0] = 1
        matrixA[i][1] = temp2[i]

    betas = np.linalg.lstsq(matrixA,matrixB, rcond=None)
    alpha = betas[0][0]
    beta = betas[0][1]

    return alpha, beta
```

Using Least Square Method





Compare

- Still different

Fourth Model

- $T_i = \alpha + \beta_1 T_{i-1} + \beta_2 T_{i-2} + \beta_3 T_{i-3} + \beta_4 T_{i-4} + \beta_5 T_{i-5} + E_i$

```
tempA = np.zeros([len(year)-5,6])
for i in range(len(year)-5):
    tempA[i][0] = 1
    tempA[i][1] = temp[i+4]
    tempA[i][2] = temp[i+3]
    tempA[i][3] = temp[i+2]
    tempA[i][4] = temp[i+1]
    tempA[i][5] = temp[i]

tempB = np.zeros(len(year)-5)
for i in range(len(year)-5):
  tempB[i] = temp[i+5]

linearFitting = np.linalg.lstsq(tempA,tempB, rcond=None)
alpha, betas = linearFitting[0][0], linearFitting[0][1:]
```

α	β_1	β_2	β_3	β_4	β_5

```
(0.014208024902743633, array([ 0.61535204, -0.00680693,  0.1022229 ,  0.24961899,  0.05804029]))
```

Fourth Model

- $T_i = \alpha + \beta_1 T_{i-1} + \beta_4 T_{i-4} + E_i$

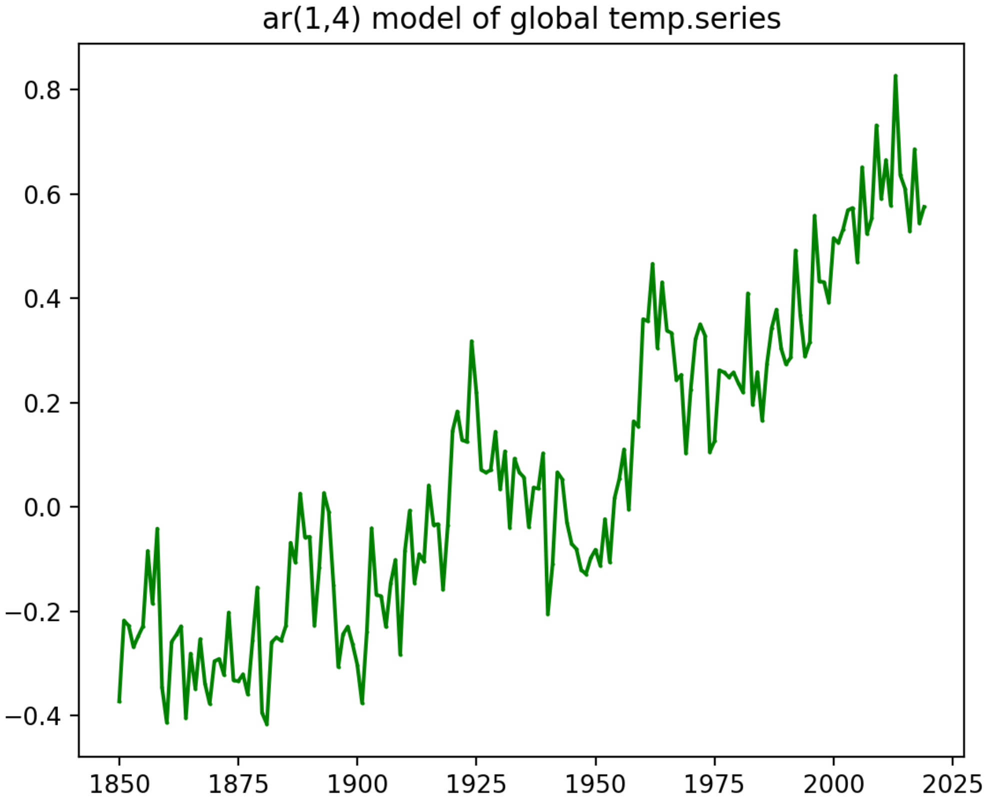
```python
tempA = np.zeros([len(year)-5,3])
for i in range(len(year)-5):
  tempA[i][0] = 1
  tempA[i][1] = temp[i+4]
  tempA[i][2] = temp[i+1]
```

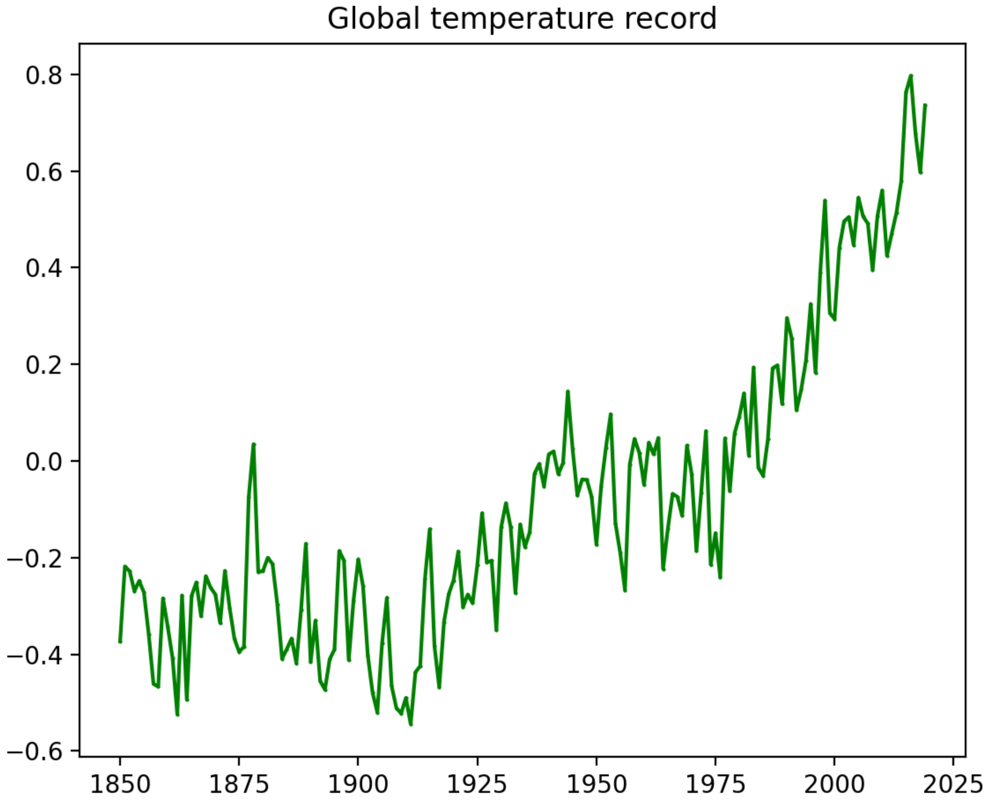
```python
tempB = np.zeros(len(year)-5)
for i in range(len(year)-5):
  tempB[i] = temp[i+5]

linearFitting = np.linalg.lstsq(tempA,tempB, rcond=None)
alpha, betas = linearFitting[0][0], linearFitting[0][1:]
```

α β_1 β_4

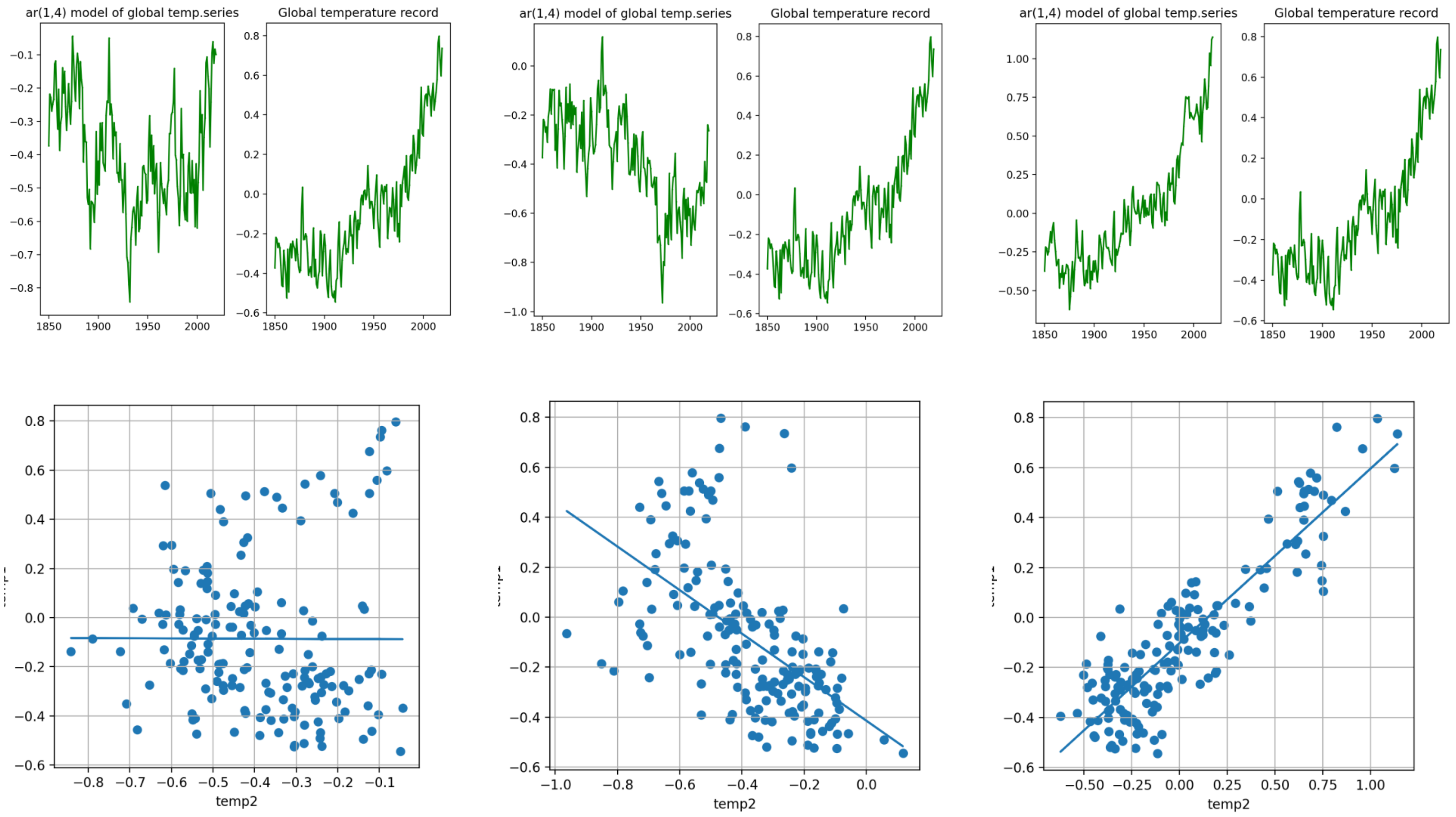
```
(0.012754473711182546, array([0.66206682, 0.34845677]))
```

If the models are very similar and the "optimal goal" is using least square to fit it and the slope would be 1. One of the run trial for this model resulting the slope to be 0.82 (which is very close)

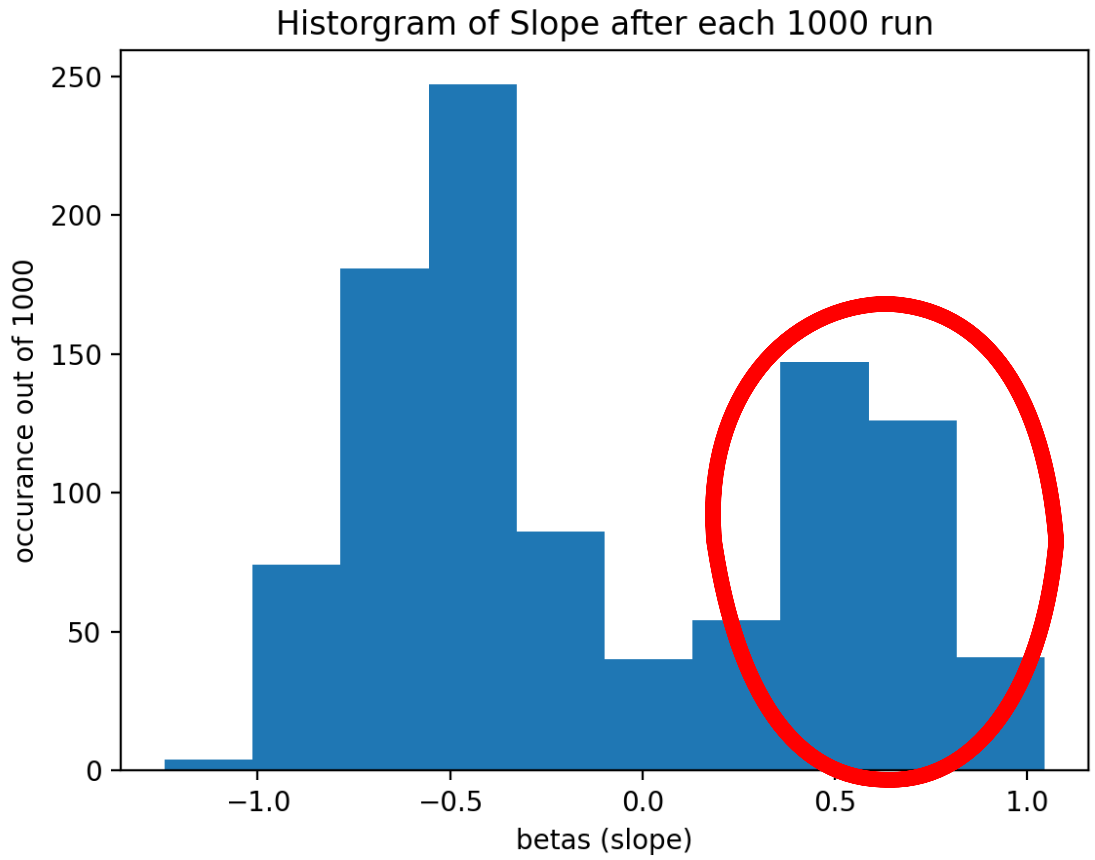




But it is not always fit



If we run this 1000 times



There is higher chance that the graph has a negative correlation with the real data compare to positive and close to 1 correlation.

Longer Temperature Record

- Through tree ring and ice core

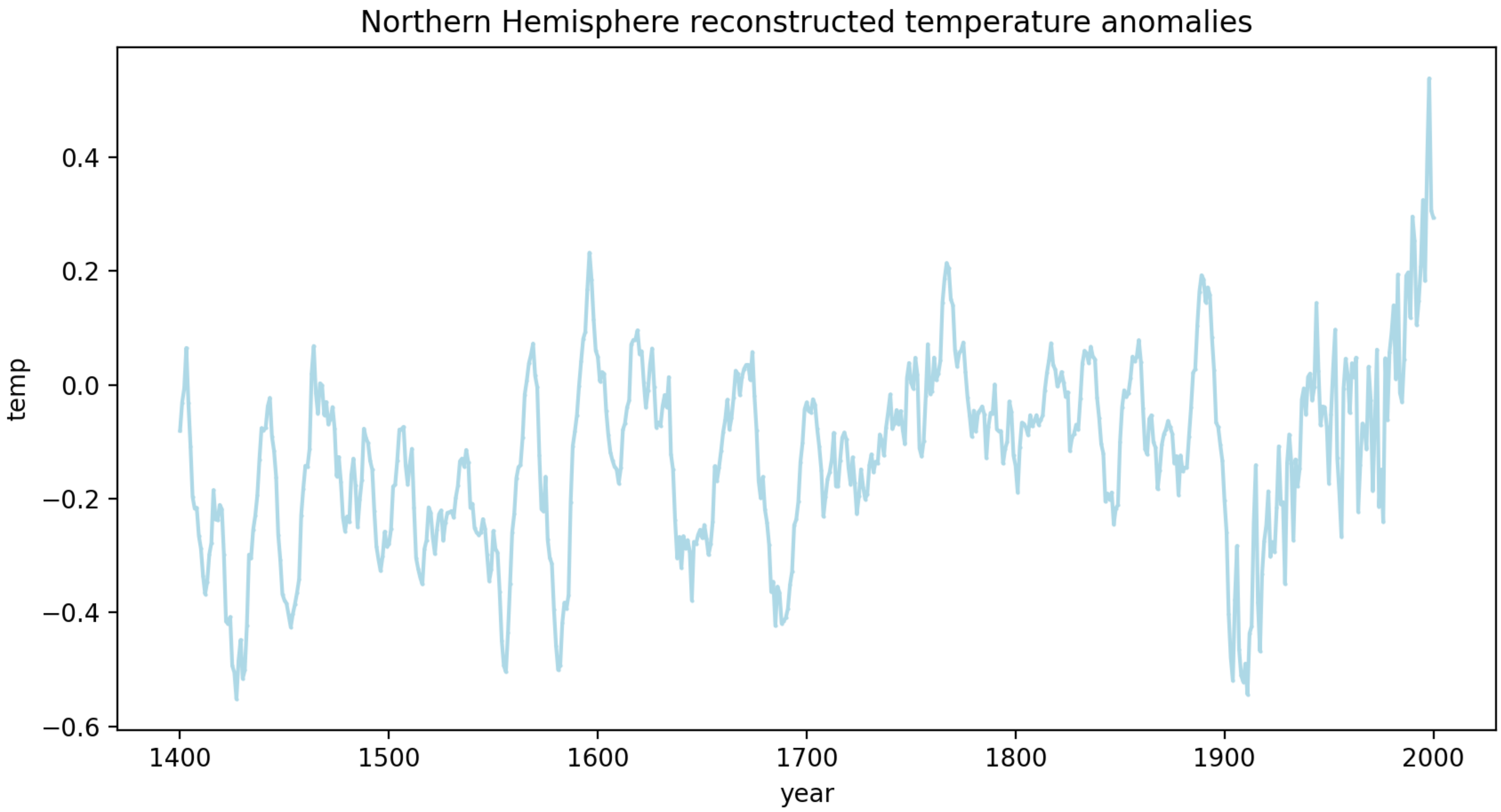
Fifth Model

- $T = \alpha + \beta w + \lambda d$

\- Find α, β and λ using the data of temperature, tree ring and ice deuterium from 1900-2000

\- Find temperature from 1400-1900

```
betas = np.linalg.lstsq(matrixA,matrixB, rcond=None)
alpha = betas[0][0]
beta = betas[0][1]
lambd = betas[0][2]
print(alpha, beta, lambd)
# Finding temperature from 1400 to 1900
e = np.random.normal(0, alpha**0.5, 501)
for i in range(500):
    prevTemp = beta*width_index[i+930-i*2] + lambd*ALL_50_full[i+108] + alpha
    # print(years2[i+930-i*2],years1[i+108])
    # print(years1[i+108])
    TEMP = np.insert(TEMP, 0, prevTemp)
    years = np.insert(years, 0, years1[i+108])
```

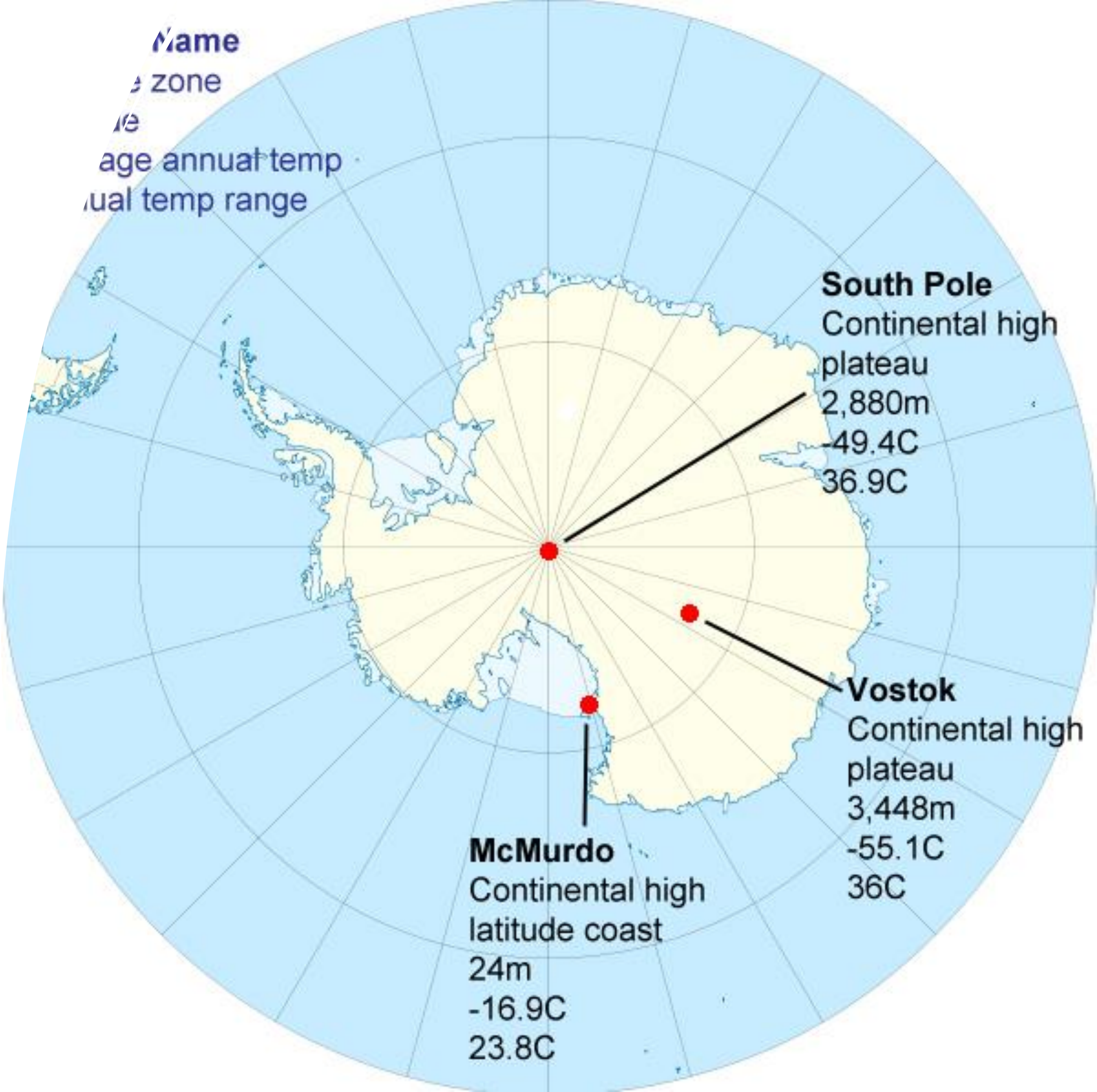

Northern Hemisphere reconstructed temperature anomalies
temp
0.4
0.2
0.0
−0.2
−0.4
−0.6
1400
1500
1600
1700
1800
1900
2000
year

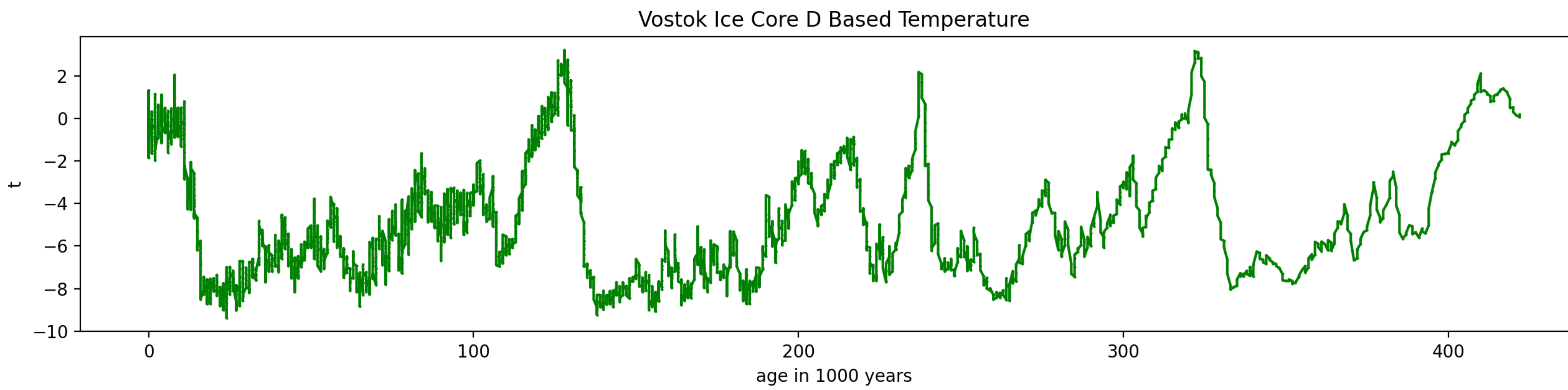
How does rising temperature related to?

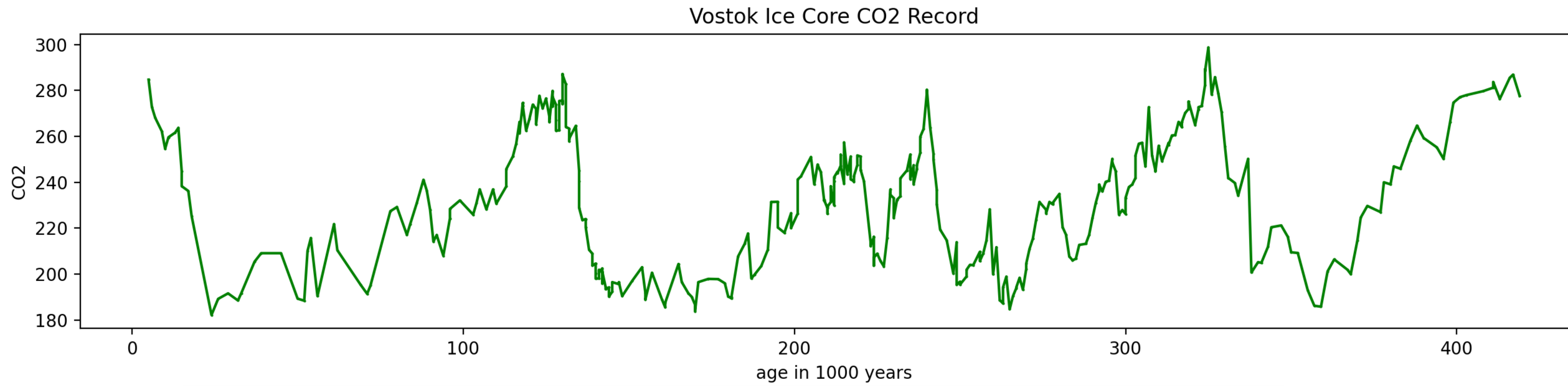
- Human activities -> Release CO_2 into the atmosphere –> the amount is more than which can be removed by natural processes
- CO_2 ”traps heat at the bottom of the earth’s atmosphere” -> rising temperature
- Skeptic: CO_2 does not change the temperature.
- Need to investigate in correlation between CO_2 and rising temperature.
- Using Vostok ice-core!

Vostok's Ice Core

- Gases can be trapped during the formation of ice.
- These gas pockets can be analyzed for its CO2 concentration content.
- Up to the last 414,000 years (1)
- Snow on ice contains record of temperature because of the formation of snow (2)

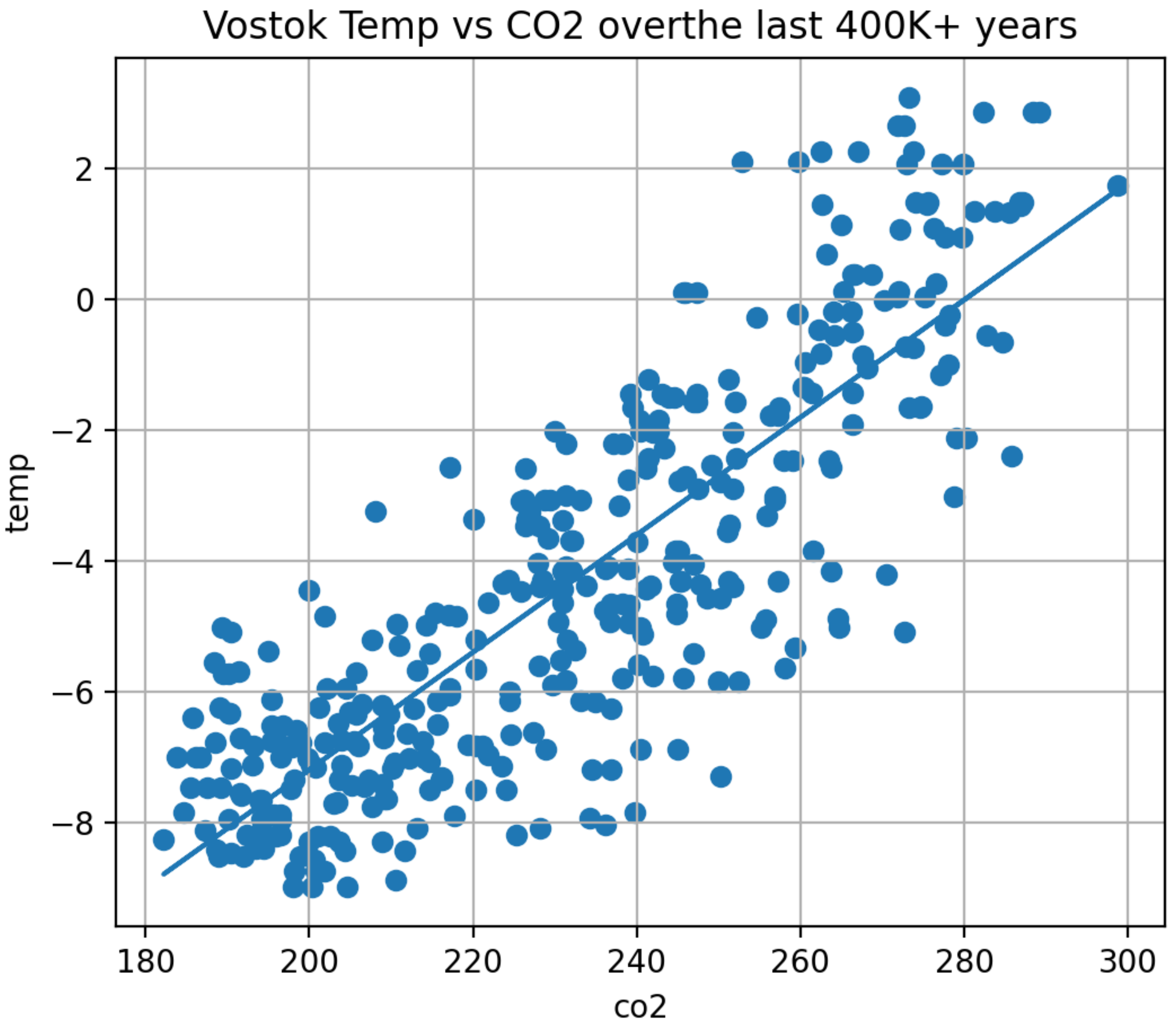



Vostok Ice Core D Based Temperature
t
age in 1000 years
0
100
200
300
400
2
0
−2
−4
−6
−8
−10


Vostok Ice Core CO2 Record
CO2
age in 1000 years
0
100
200
300
400
300
280
260
240
220
200
180

Noticing

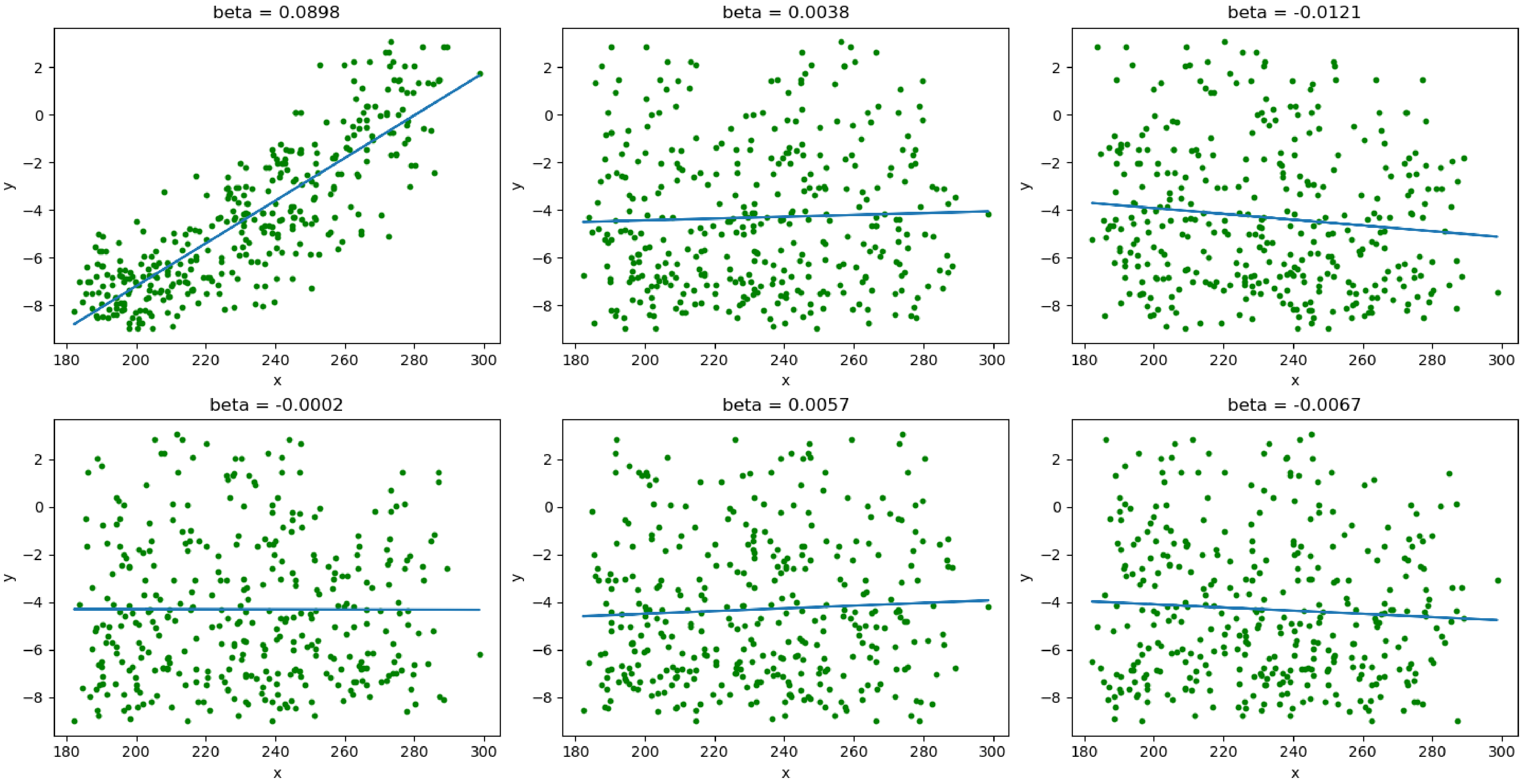
- The oscillation pattern after ~100,000 years.
- CO_2 level never past 300 ppm comparing to today 414 ppm (3)
- To view better, use linear fit by least square method to find the relationship.


Vostok Temp vs CO2 overthe last 400K+ years
temp
co2
2
0
−2
−4
−6
−8
180
200
220
240
260
280
300

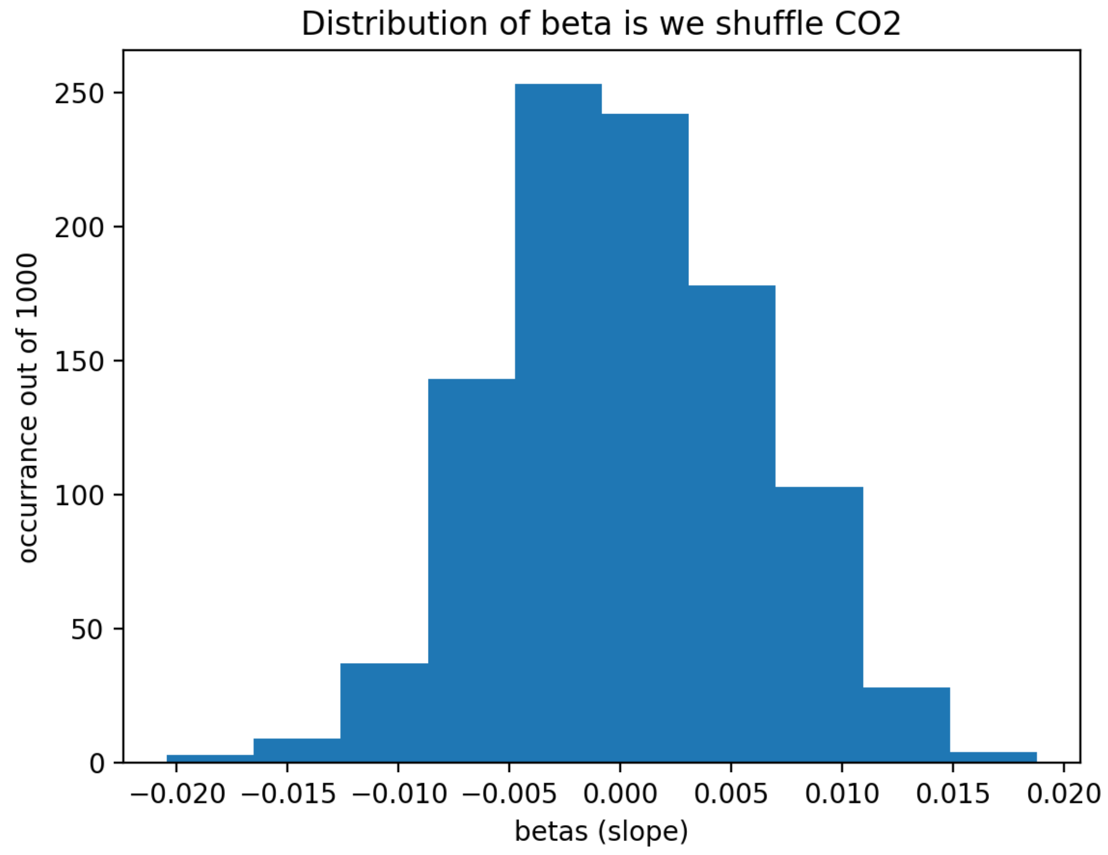
beta = 0.09

How sufficient is beta?

- Let's shuffling the co2 data to see


beta = 0.0898
beta = 0.0038
beta = -0.0121
beta = -0.0002
beta = 0.0057
beta = -0.0067
x
y

Occurrence of beta = 0.088



Conclusion

- Rising temperature is not random
- With these models, we now know a significant connection between CO_2 and the global mean temperature.
- Also means the recent abnormal increase of CO_2 (due to human activity) correlate Earth’s rising temperature.
- Plenty of models are used to simulate but failed in the long-term run, which mean human activities are the addition factor into the equation.