Investigating Climate Change Using Mathematical Modeling

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Background

Almost every year, the United Nations hosts the Climate Change Conference (COP) with leaders or representatives from several countries to review the potential implementation of the Paris Agreement 2015. In every discussion table, "Climate Change" is one of the foremost agendas. Climate Change has become a point of contention because it affects people's personal lives: their jobs, or placement., either directly or indirectly. When asking the question, "is human activity a significant contributing factor in changing mean global temperatures,"; 97 percent of the scientists and experts with a background in research and working directly with the topic agreed that human activities are the leading cause of global warming and climate change (Doran & Zimmerman, 2009). Generally, people believe that the increased amount of carbon dioxide in the atmosphere has caused climate change. The Intergovernmental Panel on Climate Change, part of the United Nations, assessed the science related to climate change, stating that the dominant effect of climate change is human activities such as burning fossil fields and other land usage (IPCC, 2021). Without human activities, the amount of carbon dioxide in the atmosphere seems to stay constant because the carbon dioxide, "the CO2 given off by decaying/burning plants or animals that have died is eventually just re-absorbed by new living plants" (Wood, 2003). The amount of carbon dioxide produced from human activities is 2-3% more than natural processes can remove. Consequently, the amount of carbon dioxide keeps increasing (Wood, 2003). However, some scientists and non-scientists do not believe that carbon dioxide is changing the climate. Mathematical models need to be built to examine the issue of climate change.

Rooting in two primary data, one is the carbon dioxide records and another of global mean temperature records, to see what is happening. First, the CO2 Record is based on the "Atmospheric Carbon Dioxide (1960-2021)" graph in Lindsey (2022) and the "Antarctic CO2 Record" in Wood (2003). In Figure 1.1, the CO2 concentration has increased by roughly 31% since 1960. A question is raised "is CO2 a cause for concern?" Carbon Dioxide is one of the greenhouse gasses that "trap infrared radiation near the surface, warming the climate." (IPCC, 2021). Second, the global mean temperature record can be obtained from the data set HadCRUT5 from Met Office Hadley Centre observations, plot the data, and we get Figure 1.2 (Morice et al., N.D.). From this figure, the temperature increased rapidly after 1920, which correlates to the rise of automobiles.

Despite evidence that the temperature and the CO2 concentration are increasing, the skeptics argue that "since climate is highly variable either way, the apparent recent upswing in the data is just one of those chance events that are likely to happen from time to time" (Wood, 2003). Moreover, the skeptics also argue that we cannot tell whether the increase in CO2 concentration causes climate change because the climate is controlled by many complex factors (Wood, 2003).

In the project's first section, mathematical models will be built to see how variable the temperature is. The second section examines the correlation between CO2 concentration and temperature.

Models

All the models are built using Python and other additional Python libraries: NumPy, matplotlib, statistics, and csv (Viet, 2022).

The first simple model, assumes the temperature equals the mean temperature plus a random variable.

$$T_i = \alpha + E_i$$

with α as the average temperature and Ei as a random variable. Estimating α and 2 by setting them respectively to the mean and variance of the data. Then, compute α, 2, and Ei to get α = -0.086 and σ = 0.307. After computing Ei and adding Ei back to the model, a graph is generated, referring to Figure 2.1. This graph greatly deviated from the Global Mean Temperature deviation (Figure 1.2).

Since the result of the first model produces too much year-to-year variation, an alternative model is needed. Assume the temperature this year equals the temperature last year plus a random variable. The second model can be:

$$T_{i+1} = T_i + E_i \ where \ E_i \sim N(0, \sigma^2)$$

This model can be called "Random Walk" because of its distinct result of wandering. From the model, we know that: Ei = Ti+1 − Ti. So, we can estimate 2 by finding the variance of the year-to-year temperature differences. We get σ = 0.116. Then, we plot and get Figure 2.2. This plot is also marginally different from the original data and does not represent the climate correctly.

The first model "produced too much year-to-year variation," while the second model "produced too little year-to-year variation" (Wood, 2003). Hence, this time an auto-regressive model is needed which is expected to produce a middle result. Assume this year's temperature is a constant plus some proportion of last year's temperature anomaly plus a random variable. Third model:

$$T_{i+1} = \alpha + \beta T_i + E_i \ where \ E_i \sim N(0, \sigma^2)$$

Since the model suggests a relationship between last year's temperature with this year's temperature, alpha and beta can be estimated. Let "temp this year" equal the set of global mean temperatures from the second to first year to the end of the set, and "temp last year" be the set of global mean temperatures starting from the first year to the second to last of the data set, a relationship can be seen from Figure 3. Estimate α and β by finding the intercept and slope by eyes, by ruler measurement, or in this case, using a built-in function from the NumPy library, polyfit():

α = intercept = -0.014, β = slope = 0.9106

There is a minor inconvenience. In this case, the usage of polyfit is not justified. polyfit() does not ensure what method is used to compute the intercept, and the slope, a usage of linear regression is needed for this problem, and the least square method would get the closest result to the data. Referring to the third model, α and β could be found from the matrix $\begin{bmatrix} \alpha \\ beta \end{bmatrix}$.

$$\begin{bmatrix} 1 & T_0 \\ 1 & T_1 \\ \vdots & \vdots \\ 1 & T_{n-1} \end{bmatrix} \begin{bmatrix} \alpha \\ beta \end{bmatrix} = \begin{bmatrix} 1 \\ T_2 \\ \vdots \\ T_n \end{bmatrix}$$

Using the function linalg.lstsq() from numpy to find alpha and beta.
α = -0.014073457932466804, β = 0.9106235027397801, σ = 0.01320336962079654
The estimate α,β, and σ will be put back into the model and run to get Figure 4.2. This graph is not closely matching with the global mean temperature data, but better in year-to-year variation. It seems to show too little correlation over the long term means that the temperature this year should not depend on just the temperature last year (Wood, 2003). To resolve that specific problem, taken five years continuously can help with the general correction with a range years, the fourth model:

$$T_i = \alpha + \beta_1 T_{i-1} + \beta_2 T_{i-2} + \beta_3 T_{i-3} + \beta_4 T_{i-4} + \beta_5 T_{i-5} + E_i$$

$$where\ E_i \sim N(0, \sigma^2)$$

Similar to the previous model, the least squares regression is used to estimate the variables α, β_1, β_2, β_3, β_4 and β_5. Modifying the previous matrices to find α and betas

$$\begin{bmatrix} 1 & T_4 & T_3 & \cdots & T_0 \\ 1 & T_5 & T_4 & \cdots & T_1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix} \begin{bmatrix} \alpha \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_5 \end{bmatrix} = \begin{bmatrix} \alpha \\ T_5 \\ T_6 \\ \vdots \\ T_n \end{bmatrix}$$

α = 0.014208024902743633, β_1 = 0.61535204, β_2= -0.00680693 , β_3= 0.1022229, β_4= 0.24961899, β_5= 0.05804029. Look at the results, it is noticeable that β_1 and β_4 are bigger than the others. So, β_1 and β_4 will "tend to dominate what the model produces" (Wood, 2003). Let's try the fifth model with just $\beta_1$and β_4, this is the model:

$$T_i = \alpha + \beta_1 T_{i-1} + \beta_4 T_{i-4} + E_i\ where\ E_i \sim N(0, \sigma^2)$$

Again, from matrices, α and β can be computed: α = 0.012754473711182546, β_1 = 0.66206682, β_4 = 0.34845677. Then plot the data and get Figure 5.1. There is a bigger temperature rise in this plot compared with the real data. This is a result of "random variation and the simple year to year correlation built into the model" (Wood, 2003) And, with this model, it takes multiple trials to get the plot which looks similar to the original data (Figure 5.2). If the models are very similar and the "optimal goal" to have is the correlation between the graphs, it has a slope of 1. Figure 5.1 has a slope of 0.82 highly close to 1. Running the model 1000 times and comparing between the model data with global mean temperature using least square method to find the line fit, there is an interesting distribution of betas (slope) in Figure 5.3. There is a higher chance that the model has a negative correlation with the real data compared to positive and close to 1 correlation. Therefore, the skeptics might be wrong as the chance of getting a plot similar to the real data is low.

Moreover, it is important to examine a longer temperature record. According to our source, temperature records prior to the middle of the nineteenth century are not available because the temperature was not measured “accurately enough, often enough, or over enough of the globe” at that time (Wood, 2003). So, “climate reconstruction” will be used. Wood states that tree growth is sensitive to temperature and “trees display clear annual growth rings” (Wood, 2003). Moreover, the ice core deuterium depends on the air temperature when the ice was deposited. As a result, tree ring width and ice core deuterium will reveal the temperature. Hence, tree ring width and ice core deuterium will be used to reconstruct the temperature. This is the model:

$$T = \alpha + \beta_1 w + \gamma d + E_i \; where \; E_i \sim N(0, \sigma^2)$$

With w is the tree ring width and d is the ice core deuterium. The least squares method is used to estimate α, β and γ. With the data of wi’s, di’s and Ti’s from 1900-2000, it is easy to get α = 5.601688247049576, β = 0.05376324751388502 , γ = 0.03844880883841319. To get the estimated temperatures from 1400-1900, put values of w_i and d_i from 1400-1900 into the equation: $T_i = \alpha + \beta w_i + \gamma d_i$, the tree ring data is from a tree on French Alps (Büntgen et al. 2012) and the ice core deuterium is from Jame Ross Island (Frank, 2010)

The temperature data from 1400-2000 is now reconstructed; see Figure 6. The pattern shows that temperature never exceeded 0.4℃ after 1900, and only in recent human time has it been above 0.2℃ and continuously increasing with no sign of going down to the sequence in the past.

The correlation between temperature and CO2 is examined in the second section of our project. Human activities realize the amount of CO2, which is more than natural processes can remove. Furthermore, CO2 "traps heat at the bottom of the earth's atmosphere," which increases the temperature (Wood, 2003). From Figure 7.1 and Figure 7.2, over the most rapid increase in CO2 concentration, the temperature also increases. Let us see how correlated is CO2 concentration and temperature.

We use NOAA Paleo Data Search to obtain data from Vostok Ice, specifically the ice core Deuterium Based Temperature (Petit et al., 1990) and ice core CO2 Concentration Record (Barnola et al., 2003). Plot and get Figure 7.1, which shows the Vostok temperature and the Vostok CO2 concentration through the years. Plot the CO2 concentration against the temperature and get Figure 7.2. Looking at the plot, there is a positive correlation between temperature and CO2 concentration. The slope is calculated as 0.09 using the same least square regression from the previous model. However, is 0.09 sufficient to say that the temperature is related to the concentration of CO2 in Earth's atmosphere? To justify the result, considering that CO2 is not related to Earth's temperature means CO2 can be at any level given at the same time. Shuffling the data and running the correlation using linear regression again 1000 times to see the distribution of the slopes see Figure 5.3. Clearly, 0.09 is out of the range, and the majority is between -0.005 and 0.005. The temperature versus CO2 is not random. Ruled out the randomness of CO2 concentration in the Earth's atmosphere and can safely prove that the temperature is directly related to the CO2 concentration.

Conclusion

In conclusion, rising temperatures are not random. With these models, we now know there is a significant correlation between CO_2 and the global mean temperature. Also, the recent abnormal increase of CO_2 (due to human activity) correlates with Earth's rising temperature. Plenty of models are used to simulate but fail in the long-term run, which means human activities are an additional factor in the equation.

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Figures

Figure 1.1. CO2 Antarctica CO2 Record

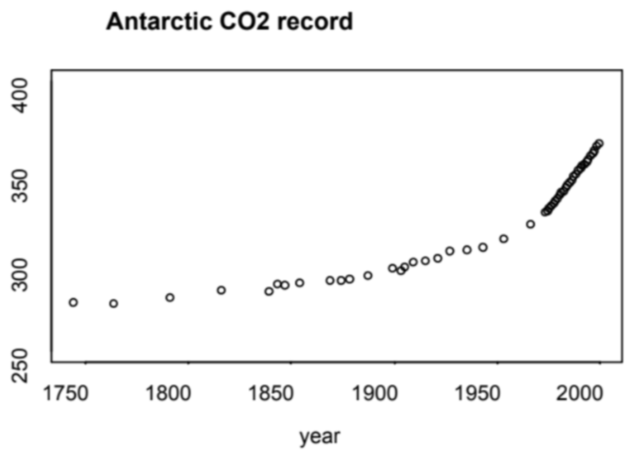


Figure 1.2. Global Mean Temperature Deviations from 1961-1990 Mean

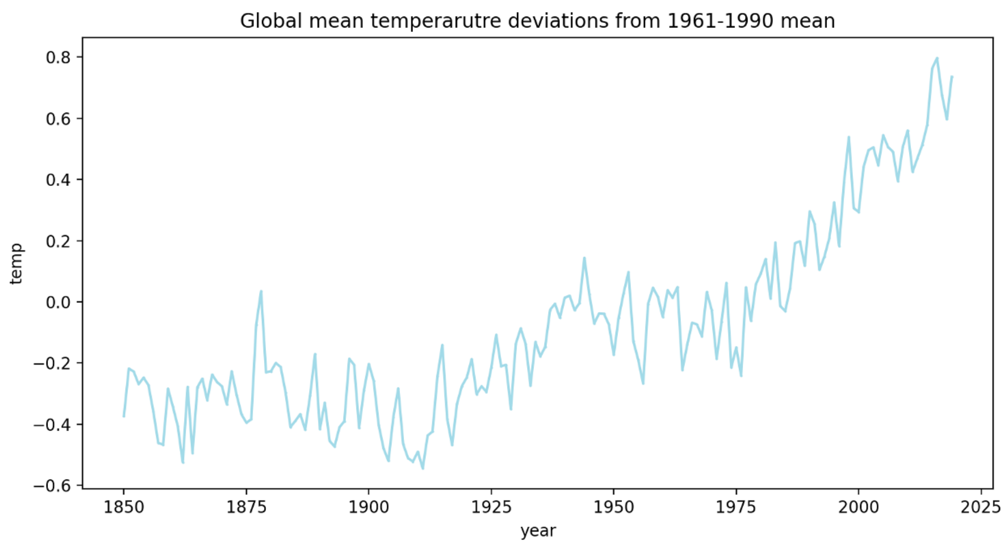


Figure 2.1. Simple, but Estimate Global Temperature Model

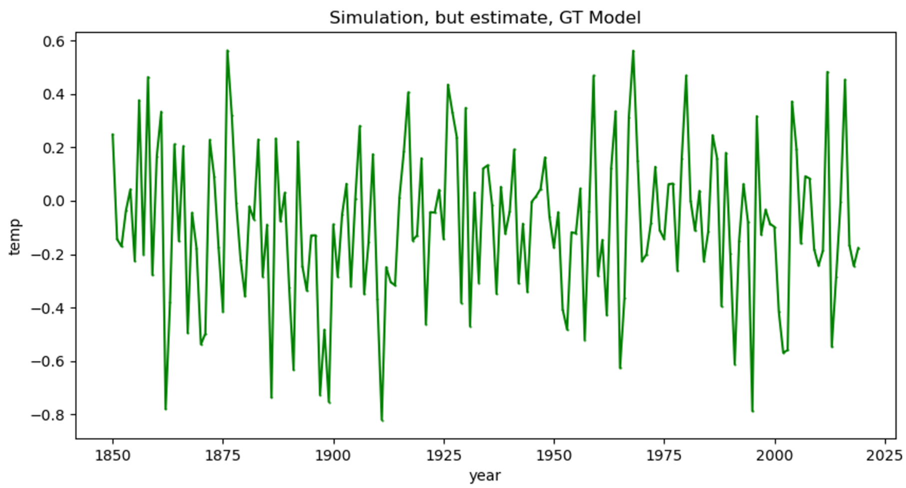


Figure 2.2. Random Walk Global Temperature Model

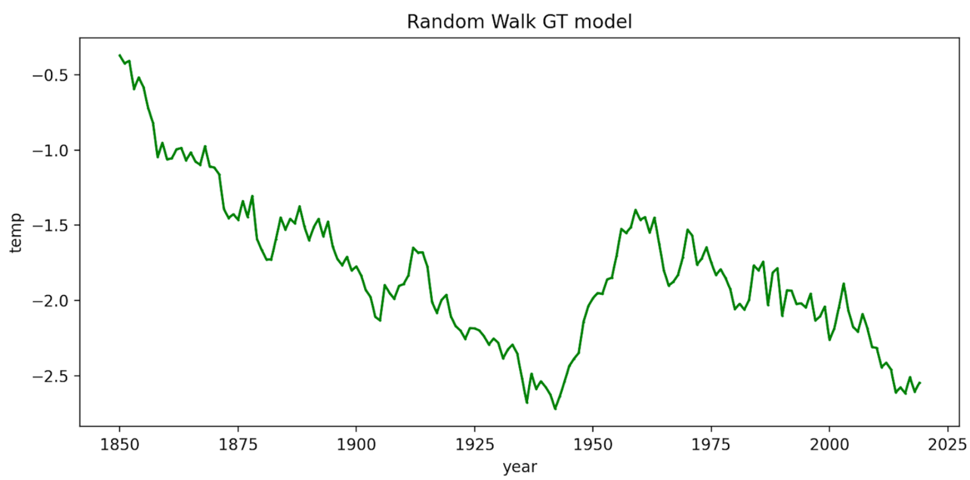


Figure 3. Line Fit This Year vs Last Year

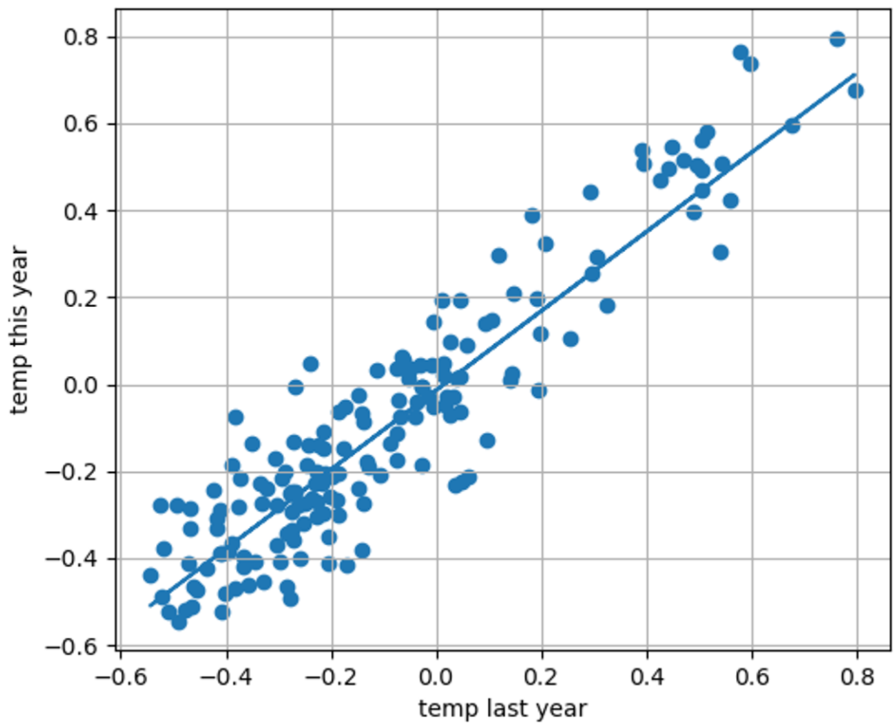


Figure 4.1. Line Fit Using Least Square Estimate

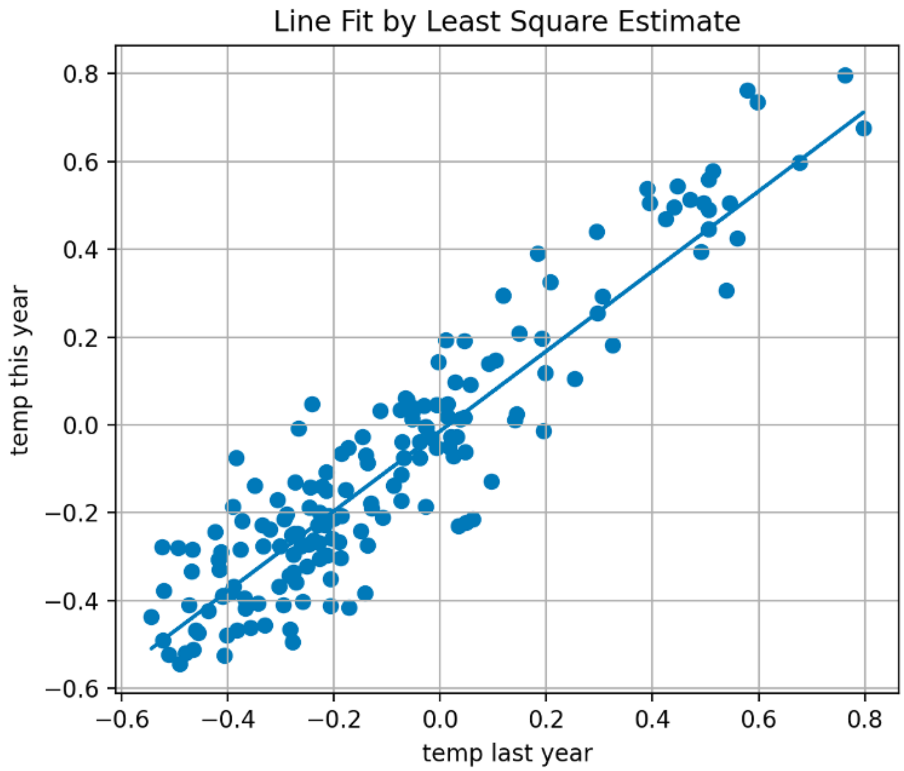


Figure 4.2. ar(1) Auto-Regressive Model

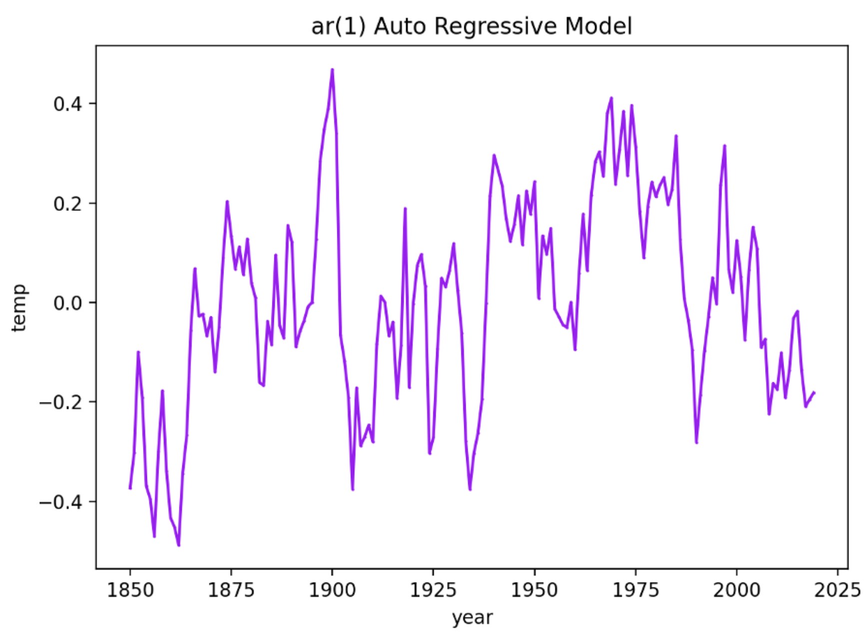


Figure 5.1 ar (1,4) Model and Real Temperature Data

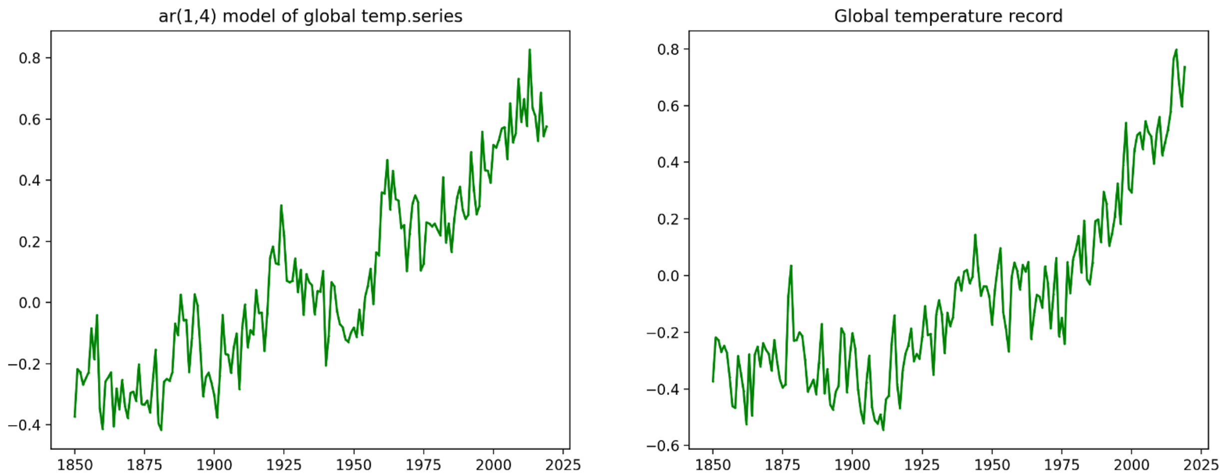


Figure 5.2

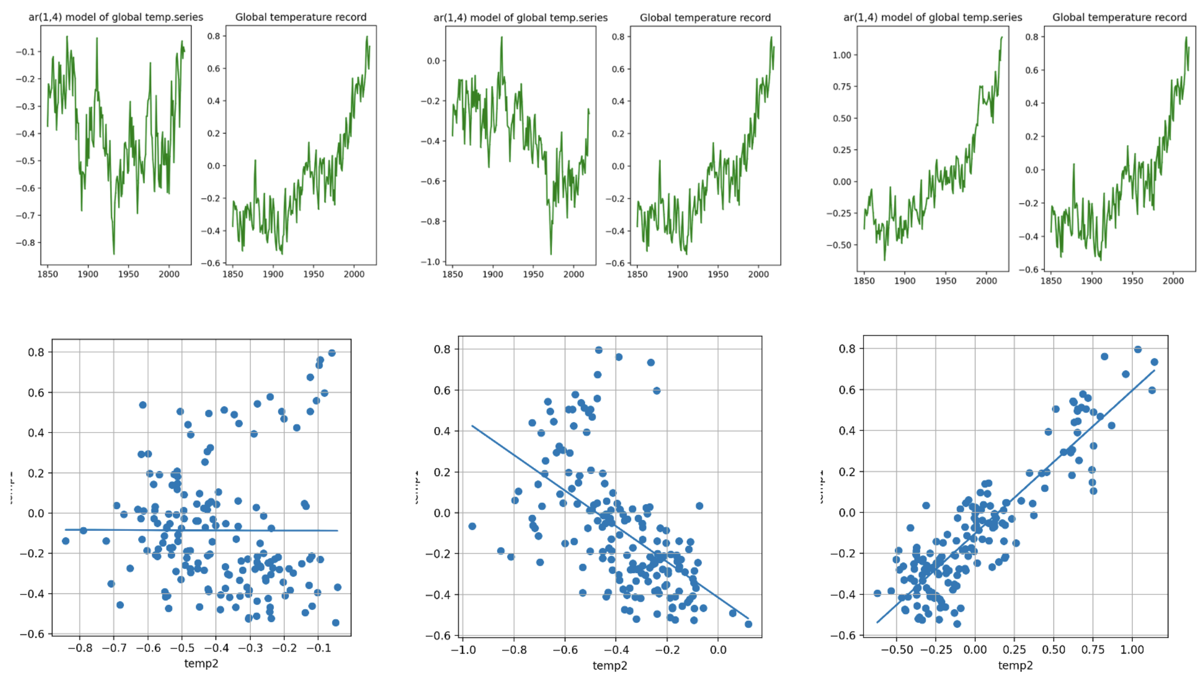


Figure 5.3

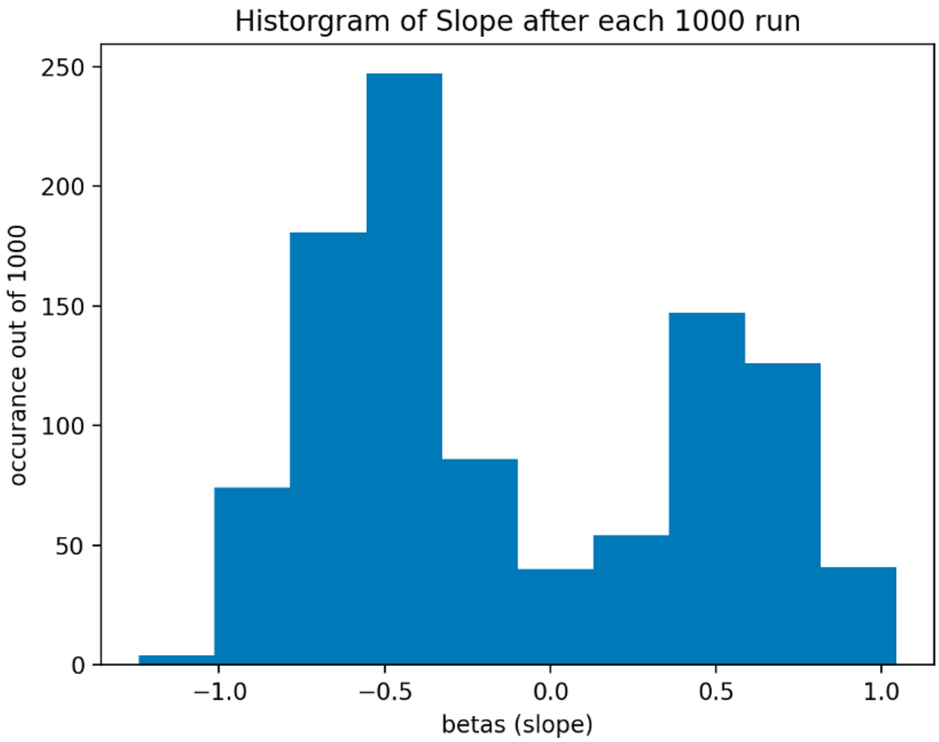

Historgram of Slope after each 1000 run
occurance out of 1000
betas (slope)
0
50
100
150
200
250
−1.0
−0.5
0.0
0.5
1.0

Figure 6. Northern Hemisphere Reconstruction Temperature

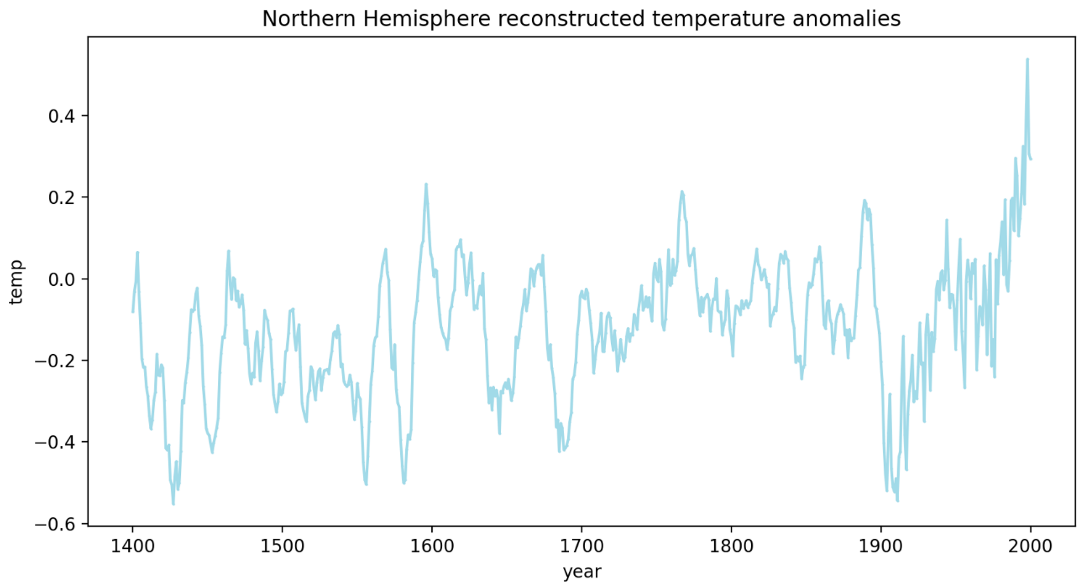


Figure 7.1. Vostok Ice Core Deuterium Temperature and Vostok Ice CO2

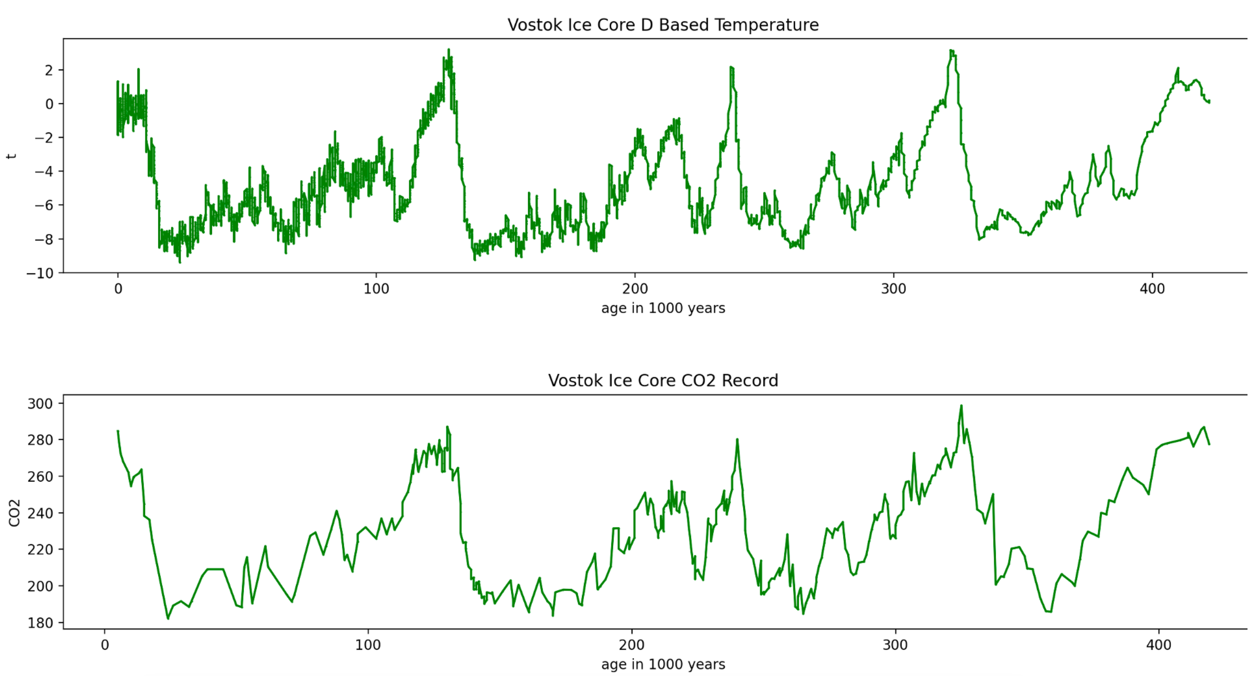


Figure 7.2. Line Fit Vostok Temp and CO2

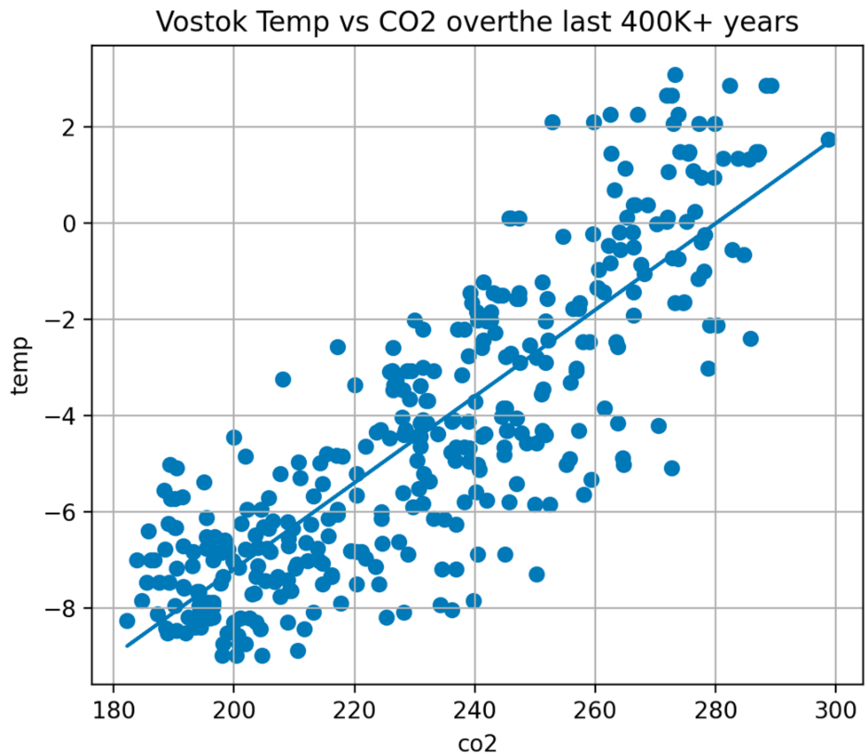


Figure 7.3.

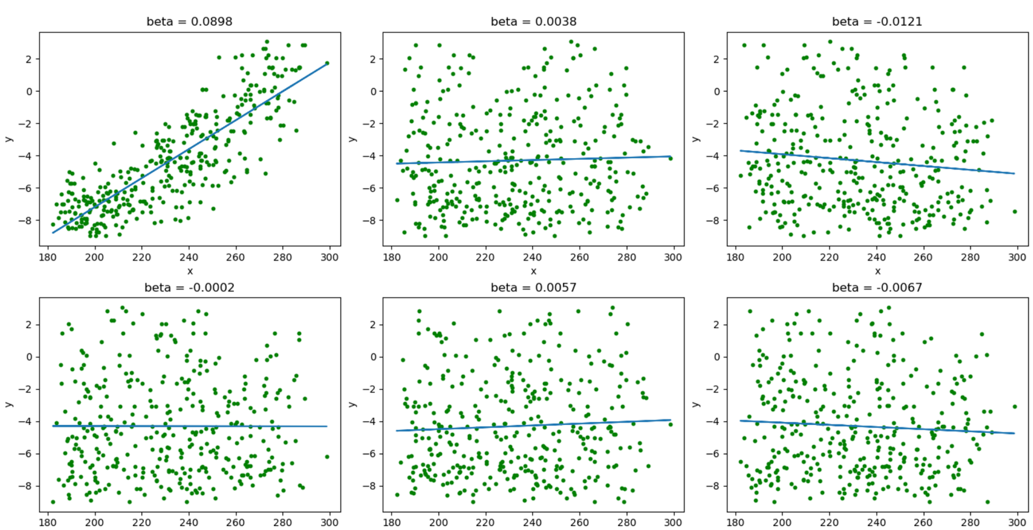
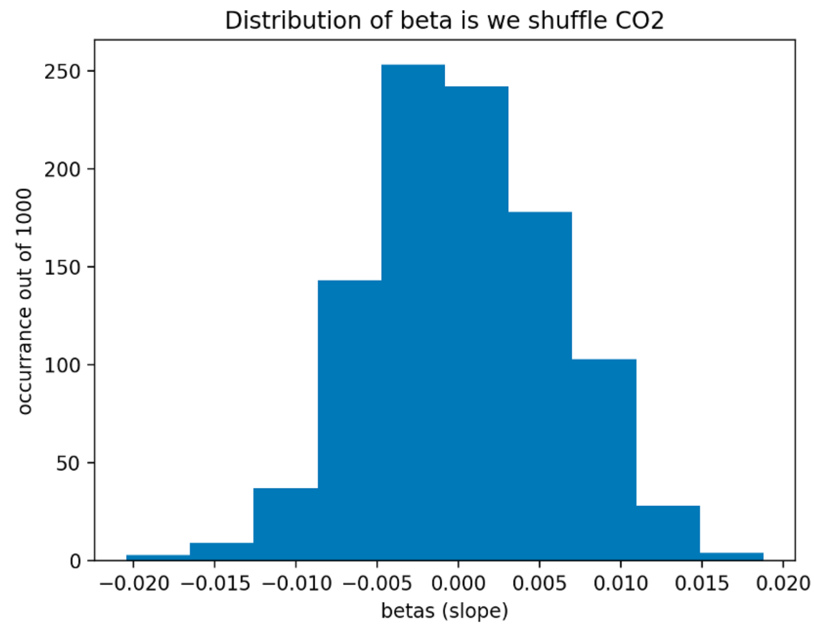


Figure 7.4.


Distribution of beta is we shuffle CO2
occurrance out of 1000
betas (slope)
250
200
150
100
50
0
−0.020
−0.015
−0.010
−0.005
0.000
0.005
0.010
0.015
0.020